

Distributed hydrological modelling: what accuracy is achievable?

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Abstract Distributed parameter watershed models that are physics-based offer distinct advantages over conceptual rainfall-runoff models. This paper describes the accuracy of simulated runoff achievable for the Illinois River (2300 km²). Parameter values estimated from physical properties of the watershed are adjusted to reproduce system behaviour. Spatial heterogeneity is accounted for at the watershed scale by maps of parameters derived from digital maps of soil properties, land use/cover, and topography. Inputs come from radar or multisensor combinations of radar, satellite, and rain gauge. Distributed hydrological models based on conservation laws have identifiable optimal values and expected behaviour and interaction during calibration. Predicted runoff accuracy for this subset is RMSE of 3.65 mm (± 1.8 mm) with the best runoff accuracy achieved to within 1.6% or 0.3 mm. For this storm subset, consistent high accuracy is achieved (<10% by volume) with an important distinction that smaller absolute differences result for larger events.

Key words Distributed Modelling Intercomparison Project; floods; hydrological modelling; hydrology; rainfall runoff

INTRODUCTION

The goal of physics-based distributed models is to represent physical processes deterministically using conservation equations at a computational element scale that in aggregate reproduces the system behaviour. One advantage, among others, is that shorter historical records are required to calibrate a physics-based distributed model (PBD) compared to long records required by conceptual rainfall-runoff models (CRR). The spatial heterogeneity of watershed characteristics is represented by PBD models using such geospatial data as digital elevation models (DEMs), land use/cover, soils, or other digital maps or databases. Such models include those proposed by Abbott *et al.* (1986a,b); Zhang & Cundy (1989), James & Kim (1990), Julien & Saghaian (1991), and Vieux & Gauer (1994). These models seek to preserve the spatial heterogeneity at the watershed scale but are lumped at the scale of the computational element, which may be a finite difference grid, stream tube, or finite element. Scientifically, PBD models are useful for investigating and testing catchments to test new hydrological hypotheses. Practically, PBD models are becoming used in operational settings (Vieux *et al.*, 2003). Unlike CRR models, PBD models have identifiable optimal parameter sets and predictable parameter interaction.

The US National Weather Service (NWS) has recently promoted distributed hydrological modelling as a major new thrust for improving flood forecasting. The Hydrology Laboratory (HL) of the NWS has just completed the first phase of the Distributed Model Intercomparison Project (DMIP). Participants download data sets and run their models in continuous or event-based simulation modes. Model simulations are compared to observed streamflow data, as well as, with simulations generated from lumped and semi-distributed applications of the Sacramento Soil Moisture Accounting (SAC-SMA) model. Using the results provided by participants of DMIP, the NWS anticipates the development of an HL distributed model that incorporates distributed modelling principles. DMIP results will be published in the *Journal of Hydrology* in the near future.

MODEL DESCRIPTION

The model used herein was first developed by Vieux (1988) and applied to a small agricultural watershed without channel routing, which was later added to account for larger watersheds with defined channels. The solution using linear, one-dimensional (1-D) elements presented by Vieux *et al.* (1990) uses a single chain of finite elements for solving overland flow. Vieux & Gauer (1994) extended this finite element solution to a network of elements representing a watershed domain with channels within a GIS. The resulting model was the distributed hydrological model *r.water.fea*, developed in 1993 for the US Army Corps of Engineers, Construction Engineering Research Laboratory, Champaign, Illinois (USA-CERL). The initial development of the model is a part of the public domain GIS called GRASS (Geographic Resource Analysis Support System). Several derivative models now exist. The *r.water.fea* model was ported from Unix to a Windows version that runs as an ArcView extension, called *arc.water.fea* (Vieux, 2001). The *r.water.fea* model accesses a map database for various parameters controlling the hydrological process. A newly developed model called *Vflo*TM implements the finite element approach, and is written in Java to take advantage of secure client-server configurations for real-time operational applications (Vieux *et al.*, 2003; Vieux & Vieux, 2002).

A mathematical analogy is derived by ignoring insignificant differential terms. The kinematic wave analogy (KWA) for overland flow is a simplification of the conservation of mass and momentum equations wherein the principle gradient is the land surface slope. The 1-D continuity for overland flow resulting from rainfall excess is expressed by:

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = R - I \quad (1)$$

where R is rainfall rate; I is infiltration rate; h is flow depth; and u is overland flow velocity. In the KWA, bed slope is equated with the friction gradient. In open channel hydraulics, this amounts to the *uniform flow* assumption. Using this fact together with an appropriate relation between velocity, u , and flow depth, h , such as the Manning equation, we obtain:

$$u = \frac{S_o^{1/2}}{n} h^{2/3} \quad (2)$$

where S_o is the bed slope or principal land surface slope; and n is the hydraulic roughness. Though not presented here, infiltration rate, I , is modelled using the Green and Ampt equation, of which saturated hydraulic conductivity, k , has a significant control. Two of the most important parameters governing response are the saturated hydraulic conductivity, k , and the roughness n . These parameters are represented as grid maps of the same resolution as the DEM. The hydraulic conductivity controls the total amount of water that will be split into the surface runoff and subsurface flow, whereas the hydraulic roughness mainly affects the peak flow and the time to peak. Model response is controlled using scalars that multiply the distributed maps of parameters. Combining equations (1) and (2) with the addition of parameter controls results in:

$$\frac{\partial h}{\partial t} + \beta \frac{s^{1/2}}{n} \frac{\partial h^{5/3}}{\partial x} = \gamma R - \alpha I \quad (3)$$

where the three scalars α , γ , and β control infiltration rate, I , rainfall rate R , and hydraulic roughness n , respectively; h is the flow depth and s is the principal land-surface slope at the centre of each grid cell. The slope and hydraulic roughness are spatially variable, while rainfall, infiltration, and flow depth are spatially and temporally variable. If we consider the rainfall as perfectly known, with no bias, then γ is equal to one. These scalars are identifiable and have defined optimal values. The inverse solution of equation (3) and the optimal values obtained through the adjoint formulation is described in Vieux *et al.* (1998) and White *et al.* (2001a,b). Model calibration is achieved through adjustment of these scalars, whether by manual or automatic means, using the OPPA methodology described by Vieux & Moreda (2002).

Finite element solution of equation (3) is accomplished by the Galerkin formulation using 1-D elements arranged in the direction of principal slope. Finite element solution details may be found in Vieux & Gauer (1994), and a more recent summary in Vieux (2001). The finite difference solution solves the resulting ordinary differential equations in time using an explicit time stepping scheme subject to the Courant-Fredericks-Levy condition.

STUDY AREA

The Illinois River watershed straddles the Oklahoma/Arkansas border, with approximately 54% of the 2400 km² draining to Tahlequah located in Oklahoma (Fig. 1). The basin is located in Delaware, Adair, Cherokee, and Sequoyah counties in northeastern Oklahoma. The average annual flow of the Illinois River as it enters Oklahoma near Watts is 20 m³ s⁻¹, which increases at Tahlequah to 29 m³ s⁻¹, after which it flows into Lake Tenkiller. Agriculture dominates the economy of the region. Forest or pasture land uses are located in upland areas, whereas cropland is concentrated in more fertile lowlands. Digital geospatial data representing the watershed are described as follows.

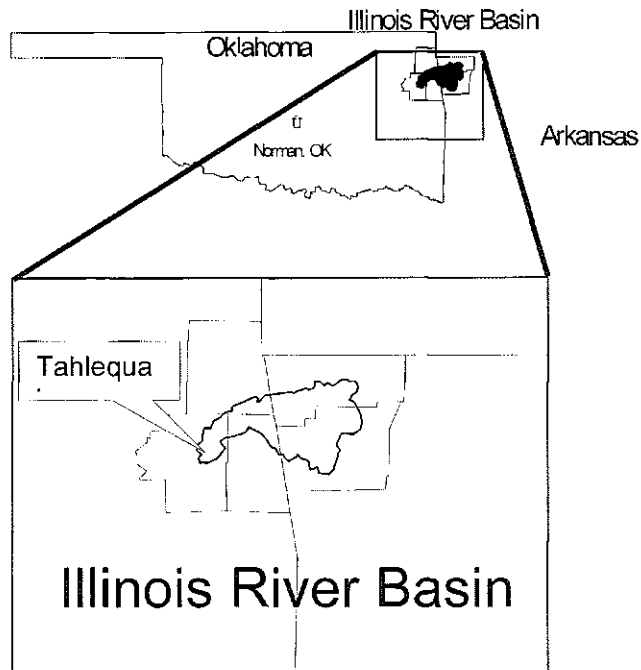


Fig. 1 Location of the Illinois River Basin in relation to Arkansas and Oklahoma.

Digital Elevation Model A 30-m resolution DEM is re-sampled to obtain a 960 m resolution map of elevation. This DEM map is used to delineate the watershed into three sub-basins. Delineation of the watershed can be done using the GRASS, *r.watershed* command or the ArcView 3.2 hydrological model extension. The *arc.water.fea* extension performs delineation and data extraction necessary for running a version of the *r.water.fea* model that has been exported to a PC such that simulation takes place on the PC in a GIS environment.

Channel characteristics Determining the exact characteristics of a channel requires extensive surveying or calibration to match hydrograph shape and peak. Trapezoidal cross-sections are interpreted from surveyed cross sections and stream gauging rating curves to obtain average values for the stream channels.

Rainfall Hourly rainfall maps as input are obtained from Stage III NEXRAD observations at 4×4 km resolution. Eight storms (see Table 1) are presented herein as described by Vieux & Moreda (2002) and in Vieux (2001). Each 960-m resolution grid cell receives rainfall from the overlying 4×4 km radar pixel. Other studies and comparison of raingauge accumulations show a mean field bias averaged over eight storms to be within 10%. No attempt to adjust the Stage-3 radar estimates are made considering that *ad hoc* adjustments to agree with raingauge accumulations are already introduced by the NWS River Forecast Centre, for which no adjustment record is generally available.

Table 1 Eight storm event summary of rainfall, simulated and observed runoff and associated errors.

Event	Rainfall (mm)	Simulated (mm)	Observed (mm)	Diff. (mm)	Abs diff. (mm)	Error (%)	(Diff.) ²
4 March 1995	30.4	4.3	9.7	-5.4	5.4	-55.7	29.20
10 May 1996	32.5	14.4	12.2	2.2	2.2	18.0	4.80
25 November 1996	53.3	23.5	29.6	-6.1	6.1	-20.6	37.20
8 June 1995	68.8	21.5	23.0	-1.5	1.5	-6.5	2.30
13 January 1995	71.4	22.9	26.4	-3.5	3.5	-13.3	12.30
4 November 1996	75.7	18.5	21.0	-2.5	2.5	-11.9	6.30
19 February 1997	76.1	29.3	33.1	-3.8	3.8	-11.5	14.40
20 April 1996	80.4	18.5	18.2	0.3	0.3	1.6	0.10
	61.1	19.1	21.7	-2.5	3.2	-12.5	RMSE = 3.65

Discharge The simulated hydrographs for each storm are compared to the USGS observed hydrographs at USGS streamgauge 07196500, Illinois River near Tahlequah, Oklahoma (see Fig. 1). Since the model simulates only the surface flow component, the baseflow separation is accomplished using the straight-line method. Baseflow components are normally less than 10% of the flow during these events. The 50% chance exceedence flow rate is approximately 30 cm.

Infiltration and roughness parameters The Green and Ampt soil parameter maps are calculated using the Brooks and Corey relationships from Rawls *et al.* (1983a,b) to estimate the infiltration parameters based on soil properties such as clay, sand content, bulk density, and pore size distribution. These soil properties are then used to estimate the Green and Ampt infiltration parameters. The soil properties for soils in the Illinois River basin are obtained from the USDA Natural Resources Conservation Service for the State of Oklahoma from county-level soil surveys. The eight storms selected had relatively dry antecedent conditions except for the 25 November–December 1996 and 4 March 1995 events. Wet conditions were assigned a nominal 70% degree of saturation, whereas dry conditions were assigned 30%. The Manning parameter maps are obtained by relating land use/cover (LULC) maps to the corresponding roughness coefficients estimated according to the dominant LULC classification.

MODEL CALIBRATION

Calibration proceeding along the lines of OPPA, adjusts initial parameter maps by a scalar in order to minimize an objective function. Hydraulic conductivity is adjusted to achieve a volume match followed by adjustment of overland hydraulic roughness affecting peak discharge and timing. Subsequent adjustment of channel hydraulics principally affecting the time-to-peak agreement with observed. Similarly, hydraulic roughness is derived from land use/cover maps. Scalars are multiplied by these parameter maps to adjust the value in each grid cell while preserving the spatial heterogeneity. The sequence of adjustment is to first minimize the objective function for volume, and then peak, obtaining an overall *optimal* parameter set for the eight storms.

RESULTS AND DISCUSSION

Results for the eight storm events are presented in detail in Vieux (2001) and Vieux & Moreda (2002). A current effort to extend these results to 22 events is underway within the DMIP project. Initial results show that only minor adjustment to hydraulic conductivity resulted when additional storm events are added to the calibration suite. The results for the eight-storm suite are described as follows.

Hydraulic conductivity is used to adjust volume, after which, hydraulic roughness is used to adjust timing and peak. Several adjustments were made to ensure that timing controlled by channel hydraulics produced consistent results at an interior point located near Watts, Oklahoma. Calibration is performed for the suite of eight events by adjusting parameter maps by scalar multipliers (α and β in equation (3)) that minimize volume and peak discharge errors. Table 1 reports the results for each of the eight storms along with a summary (shown in bold on the bottom line of the table) for the suite of eight storms. The second column indicates the rainfall depth averaged over the watershed. The third and fourth columns are the runoff volumes in watershed depth (mm) for simulated and observed amounts, respectively. The fifth and sixth columns are the runoff differences and absolute runoff differences reported as mm of simulated and observed runoff. The seventh column is the simulated error reported as a percentage of the observed runoff. The final and eighth column is the square of the error, which when summed, is used to compute the root mean square error (*RMSE*) shown in the last line of the column.

Several interesting features emerge when considering the absolute difference between simulated and observed runoff. The absolute differences expressed as an equivalent watershed depth (mm) are all 6.1 mm or less, averaging 3.2 mm. The average rainfall is 61.1 mm. Using a criterion that simulated runoff should be within 10% of the average rainfall, then performance of the model for any one storm is 10% or better. If calibration were performed with any one storm, the simulated output would be no more than 10% of the input. Expressing the simulated runoff as a percentage error, the range of error is more variable. The 4 March 1995 event was the smallest event with 30.4 mm of rainfall.

The simulated runoff was 4.3 mm compared with 9.7 mm observed, or a 55.7% error. The model tends to predict runoff consistently to less than 6.1 mm, with an overall performance of *RMSE* of 3.65 mm (± 1.8 mm). The relatively consistent prediction is evident in Fig. 2 where the rainfall and simulated runoff difference (Diff. in Table 1) are shown as vertical bars. It appears from this storm suite that there may be a tendency to more accurately predict with larger rainfall events. For example, the model predicted to within 1.6% or 0.3 mm for the 20 April 1996 event; the best accuracy achieved. While rainfall more than doubles from 30.4 mm to 80.4 mm, the runoff prediction errors remain consistently below 6.1 mm.

Considering the uncertainties in the radar rainfall inputs, the simulated volumes agree remarkably well with those observed. Removal of systematic error (bias) found in radar rainfall estimates is accomplished by adjusting the radar amounts to agree with raingauge accumulations. The adjustment procedure, called P1, performed by the NWS approaches bias correction by sampling the mosaic of hourly rainfall accumulation and comparing with raingauge amounts for the same time period. The radar/gauge ratio

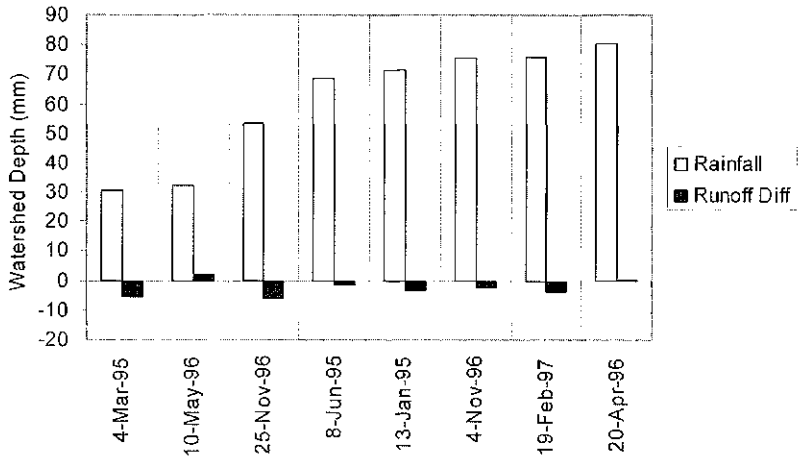


Fig. 2 Eight storm rainfall and simulated runoff differences expressed as an equivalent depth distributed over the watershed.

at each gauge location is then contoured with some smoothing and quality control on non-reporting gauges or other anomalies. Using these ratios, the radar is adjusted such that it agrees with raingauge accumulations. A limited comparison of storm totals for the eight storms revealed an average bias over the eight events to be less than 10%. Errors in runoff simulation and radar inputs for individual storm events are more or less than the average. Given this uncertainty in the input to the model, it is unlikely to obtain simulated volumes that are better than 10% error. The overall simulation accuracy for the eight-storm event suite has a regression coefficient of $R^2 = 0.87$ and a slope of 1.12, indicating a systematic bias towards underprediction. Figure 3 depicts a hydrograph occurring on 19–25 February 1997, selected from the 22 storm events simulated as a part of DMIP showing good agreement in this case. Because optimal parameters were selected that minimize error for the eight events, agreement can be improved on an individual event basis.

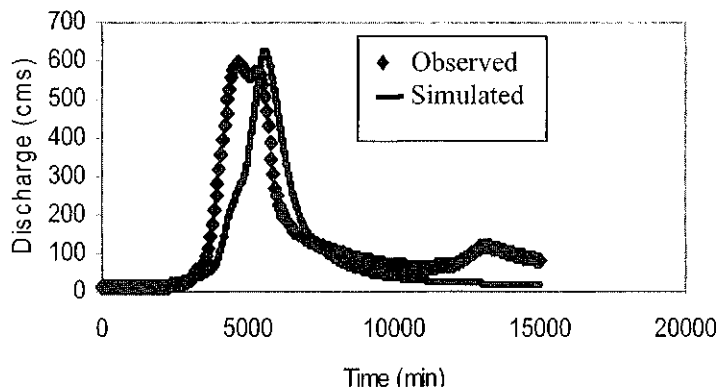


Fig. 3 Simulated and observed hydrograph selected from the 22 DMIP events (19–25 February 1997).

CONCLUSIONS

The results of this calibration are encouraging given that several storms during different seasons were well reproduced with high accuracy. Because of the consistent behaviour, the optimal parameter set would be chosen regardless of which storms were used. The accuracy achieved in runoff volume for all storms is 6.1 mm or less, with an *RMSE* of 3.65 mm. Additional events have been simulated for a total of 22 events in the DMIP experiment. Slight changes resulted, but did not materially change the calibration. Seasonality effects related to land use/cover, as well as radar rainfall bias or anomalies, decrease simulation accuracy. The optimal parameter set is stable and did not materially change as more events from different seasons or years are added. Further, calibration using any one of the events would have yielded about the same result. In no case did the simulated runoff exceed 10% of the rainfall input. An important behaviour of the model is that the absolute difference between simulated and observed runoff was the smallest for the larger events, which typically cause the most flooding.

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