

Improving *a priori* estimates of hydraulic parameters in a distributed routing model via variational assimilation of long-term streamflow data

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Abstract Due to very large dimensionality of the problem, exhaustive estimation of grid cell-specific parameters in distributed models is highly impractical (if not infeasible). As such, much attention has been given in recent years to accurate *a priori* estimation of distributed parameters from all available data sources, to the degree possible. Though they may capture the spatial variability of the parameters reasonably well, such *a priori* estimates of distributed parameters are subject to various sources of systematic and scale-dependent errors that can potentially be of rather large magnitude. The purpose of this work is to explore improving, or refining, *a priori* estimates of distributed hydraulic parameters of a kinematic wave routing model via variational assimilation of long-term streamflow observations.

Key words distributed parameters; hydraulic routing; streamflow; variational assimilation

INTRODUCTION

With the wide availability of digital elevation models (DEM), various spatial data sets and GIS tools, one can now routinely produce *a priori* estimates (whatever the goodness and physically-basedness may be) of grid-specific hydraulic parameters in distributed routing models. Though they may capture the spatial variability of the parameters reasonably well, such *a priori* estimates are necessarily subject to various sources of systematic and scale-dependent errors that can potentially be of rather large magnitude. Accordingly, if viable techniques can be found, it may be possible to significantly improve the goodness of the *a priori* estimates with only a modest amount of computational expense. Here, we explore variational assimilation, i.e. gradient-based least squares minimization based on adjoint of the (forward) dynamic model under consideration, as a potential tool for improving, or refining, the *a priori* estimates of distributed routing parameters. The reasoning behind the use of variational assimilation here is that, if the model dynamics is realistic and the linearized dynamics does capture the sensitivity of the distributed parameters to the observed streamflow at the basin outlet, the adjoint technique offers a practical way of identifying the parameters (and their locations) that exert the greatest influence on the streamflow simulation, and of quantifying, under the least squares criterion, the magnitude of adjustment necessary.

ROUTING MODELS

The routing models considered are the kinematic wave formulations for hillslope and channel flows as implemented in the US National Weather Service Hydrology Laboratory's (NWS/HL) Research Modeling System (RMS; Koren *et al.*, 2002). Soil moisture accounting is carried out, as in RMS, by a gridded application of the Sacramento model (Burnash *et al.*, 1973) with no lateral exchange. Below, we reproduce the description of the routing models from Koren *et al.* (2002).

The hillslope flow is modelled as:

$$\partial h / \partial t + L_h \partial q / \partial x = R_s, \quad 0 \leq x \leq L_h \quad (1)$$

$$q = q_s h^{5/3}, q_s = 2DS_h^{-1/3} n_h^{-1} \quad (2)$$

where h is the depth of the overland flow, q is the discharge per unit area of hillslope, R_s is the fast-responding runoff from the soil moisture accounting model, S_h is the hillslope, n_h is the roughness coefficient, D is the drainage density ($1/[L]$) and L_h is the hillslope length ($= 1/(2D)$). The three parameters, D , S_h and n_h , in q_s are specified for each grid cell. A no-flow boundary condition is assumed at $x = 0$ and L_h .

The channel flow is modelled as:

$$\partial A / \partial t + \partial Q / \partial x = (q_{Lh} + R_g) f_c / L_c, \quad 0 \leq x \leq L_c \quad (3)$$

$$Q = Q_s A^{M_c} \quad (4)$$

Where A is the (wetted) channel cross section, Q is the discharge, q_{Lh} is the lateral inflow per unit length of the channel, R_g is the slow-responding runoff from the soil moisture accounting model, f_c is the area of the grid cell, L_c is the channel length within the grid cell, Q_s is the channel specific discharge, and M_c is the exponent parameter. The upstream boundary condition at each grid cell is the total discharge draining into it. Assuming the power-law relationship of $W = a H^b$ between the top width W and depth of the channel H , where a and b denote the width and shape parameters, respectively, the *a priori* estimates of Q_s and M_c are given by:

$$Q_s = S^{1/2} n^{-1} (1+b)^{-2b/(3(b+1))} a^{-2/(3(b+1))} \quad (5)$$

$$M_c = (b + 5/3)/(b + 1) \quad (6)$$

The parameters to be adjusted are then the specific hillslope discharge, q_s , the exponent, M_c , and the channel specific discharge, Q_s .

PARAMETER ADJUSTMENT

To refine the *a priori* estimates, we solve the following least squares minimization problem subject to the routing model dynamics (equations (1)–(4)):

$$\text{Minimize } J = 1/2 [Z_Q - H(X)]^T R^{-1} [Z_Q - H(X)] \quad (7)$$

In the above, Z_Q denotes the vector of long-term streamflow observations, H denotes the structure function that relates the distributed routing parameters to the streamflow at the basin outlet, X denotes the vector of all three routing parameters at all grid cells,

and \mathbf{R} denotes the covariance matrix associated with the error, $Z_Q - H(\mathbf{X})$. Assuming temporal independence among the errors, we may rewrite equation (7) as:

$$\text{Minimize } J = 1/2 \sum \{z_{Q,i} - h_i(\mathbf{X})\}^2 / \sigma_Q^2 \quad (8)$$

In the above, $z_{Q,i}$ denotes the streamflow observation for the i th hour, $h_i(\mathbf{X})$ denotes the structure function that relates the routing parameters to the simulated streamflow at the basin outlet at hour i , σ_Q^2 denotes the error variance (assumed to be flow magnitude-independent), and the summation is over all hours in the duration of the streamflow record of choice. In equations (7) and (8), the absence of the background error term (which levies penalty for adjusting the *a priori* estimates “too much”) is a reflection that we have little *a priori* knowledge as to how “good” the *a priori* estimates are.

The minimization is carried out using the conjugate gradient method of Fletcher-Reeves-Polak-Ribiere (Press *et al.* 1986). The gradients necessary for the minimization were evaluated via the adjoint of the routing models, equations (1)–(4), generated by the Tangent linear and Adjoint Model Compiler (TAMC; Giering, 1999). Significant modifications were necessary to the automatically generated adjoint code to improve computational efficiency.

RESULTS AND DISCUSSIONS

To test the procedure, the *a priori* estimates of routing parameters (see Koren *et al.*, 2002 for details) for the basin near Watts, Oklahoma, USA, were refined based on a 3-year period of hourly streamflow observations. Extensively modelled in the Distributed Model Intercomparison Project (DMIP, <http://hsp.nws.noaa.gov/oh/hrl/dmip>), the basin has 100 grid cells, the size of the cell being approximately $4 \times 4 \text{ km}^2$. The total number of control variables in the minimization was hence 300. Figure 1 shows the scatter plots of the parameters Q_s and M_c between the *a priori* and the refined estimates. Note that a uniform *a priori* estimate was used for m . Sensitivity to the hillslope specific discharge, q_s , was found to be rather small in that the adjusted estimates differ little from the *a priori* estimates (not shown). The channel specific discharge, Q_s , tends to be larger in the upstream areas due to higher slopes (not shown) hence the scatter plot of Q_s in Fig. 1 is an indication that the streamflow at the outlet is more sensitive, as one might expect, to the channel-specific discharge along the main stem closer to the outlet than at the upstream branches. The scatter plot of Q_s also indicates that, in the upstream areas where the parameter exerts little influence to the outlet streamflow, there is little point, as one might also expect, in adjusting the *a priori* estimates in the absence of any other observations. Figure 2 shows the scatter plots of streamflow between the observed and the simulated flows, before and after the adjustment over the 3-year parameter estimation period. A significant improvement in streamflow simulation at the outlet is apparent. In addition to independent validation of the simulation at the outlet, which is under way, a true test of the procedure is to perform independent validation of streamflow at interior locations. For the parameter adjustment procedure to be operationally viable, flow simulations at interior locations based on adjusted parameters should consistently be at least as good as those based on the *a priori* estimates. Such a “blind” independent validation is also under way, and the results will be reported in the near future.

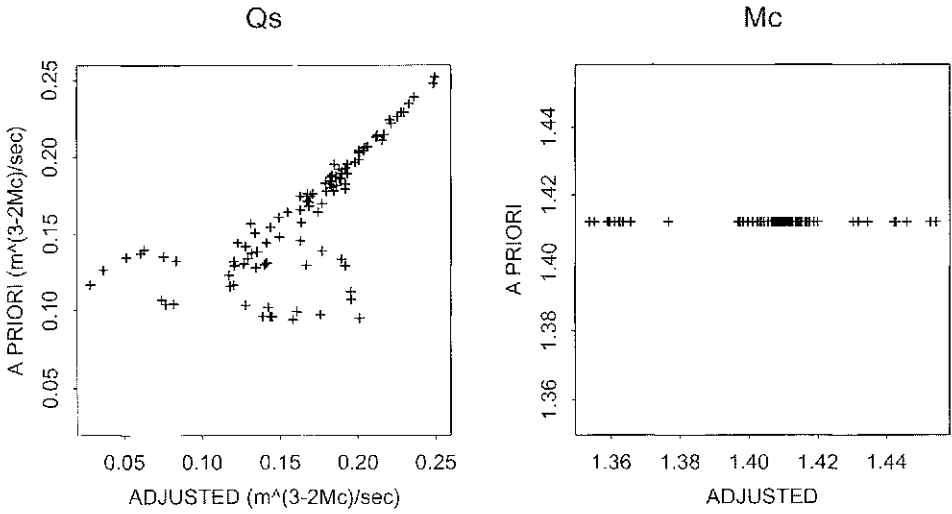


Fig. 1 Scatter plots of distributed routing parameters; the channel-specific discharge, Q_s , and the exponent, M_c , at all grid cells between the *a priori* and the adjusted estimates.

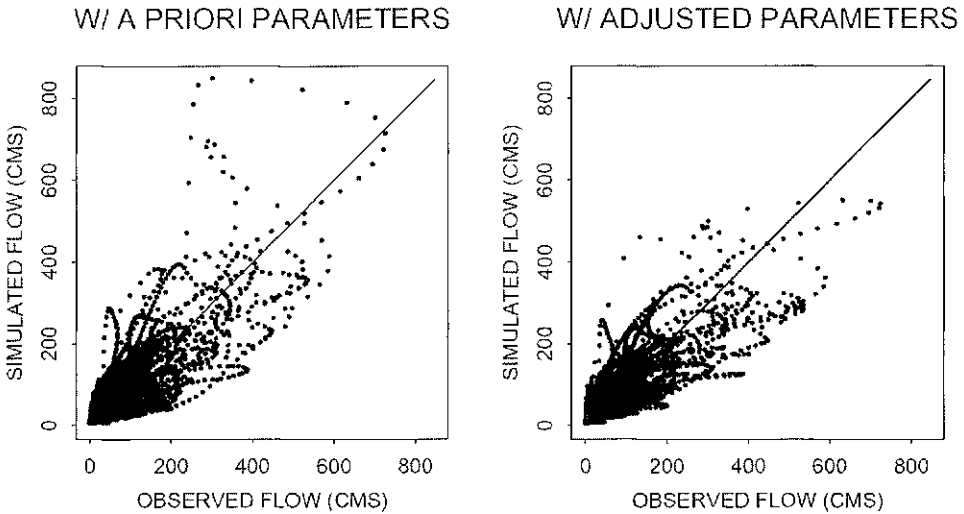


Fig. 2 Scatter plots of hourly streamflow at the outlet between the observed and the simulated flows before and after the adjustment.

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