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On the modelling of temporal correlations in spatial-cascade rainfall downscaling

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Abstract Multiplicative cascade schemes are extensively used to distribute the large-scale forcing of precipitation imposed by global atmospheric models for a limited spatial domain into its constituent grids. A model is proposed to introduce temporal persistence to one of the cascade based rainfall disaggregation models used for this purpose. A time series scheme based on the Markov process is used to relate the cascade properties to those of the previous time step. The resulting distributions could mimic the pixel level temporal persistence of spatial rainfall effectively, making cascade distributed spatiotemporal rainfall more suitable for applications in surface hydrology.

Key words multiplicative cascade scheme; rainfall disaggregation; rainfall downscaling

INTRODUCTION

Modern-day hydrologists involved in solving diversified water resources and environmental issues are increasingly recognizing the importance of incorporating high resolution spatial and temporal variability of rainfall as one of the most important parameters in applied hydrology applications. Spatial measuring techniques like weather radar reveal a much more complicated picture of rainfall variability than those revealed by traditional gauge networks. In spite of being complicated, the understanding of the relationships between different spatial scales has become crucial for the successful utilization of available spatial rainfall data sources for hydrological problem solving. One such area is the resolving of the aerial distribution of rainfall under specified forcing conditions. This particular scaling problem finds its uses in many areas including: (a) utilization of global and regional weather forecasts as data sources for locations that do not have adequate local monitoring facilities; and (b) understanding of the consequences of global climatic changes as predicted by global climatic models on the local hydrology. Perhaps the most widely researched solution for the problem is the use of limited-area atmospheric model outputs with dynamical equations (Pielke, 2002). Downscaling model outputs based on stochastic properties provide much simpler and computationally efficient solutions without the physical basis. There are also hybrid models that represent a compromise between the physical basis and simplifications based on stochastic models (e.g. Georgakakos & Krajewski, 1996). The present research using the fractal theory falls into the category of stochastic models.

Using cascade theories to spatially distribute global or mesoscale rainfall has a number of attractive features, where the adequacy of the ability to mimic the spatial variability and discontinuity as in observed rainfall is a major one. In the last two decades, a number of multiplicative cascade-based rainfall downscaling models and techniques have been developed. However, in spite of these numerous advances, the applications of these products in hydrological applications like runoff studies have been rare. Most of the successful efforts of multifractal analysis and cascade modelling of rainfall have been limited to either temporal or spatial dimensions. On the other hand, even a crude maintenance of both spatial and temporal correlations is a prerequisite for distributed run off modelling. Hydrological processes such as infiltration, evapotranspiration and overland flow critically depend on the temporal distribution of a rainfall event, in addition to the quantity of precipitation. Due to this sensitivity of the governing equations of watershed hydrology to the memory of the past states, a rainfall product needs to be accurate in the temporal persistence, in addition to the spatial features, for it to qualify as a candidate for surface response studies. The effects of temporal correlations can be relatively small at temporal scales like a day or a week, due to the smaller temporal limits of the prevailing rainstorm evolution. However, at hourly or smaller scales the effects of rainfall evolution in time are predominantly exhibited. Thus, the unavailability of means to maintain the temporal persistence in spatial models is perhaps a main reason for the lack of appeal to apply cascade downscaling of rainfall to runoff and other hydrological studies.

One theoretically attractive way of overcoming this inadequacy is to model rainfall as a full spatiotemporal process (Pegram & Clothier 1999; Deidda, 2000) utilizing the famous Taylor hypothesis. This involves transformation of the temporal dimension into a third spatial dimension by multiplying with a hypothetical velocity factor, derived empirically from observed rainfall. On the other hand, Lammering & Dwyer (2000) proposed an empirical scheme where the early steps of cascade simulations are exactly duplicated in the successive time steps. Further, they showed that the introduction of even a crude representation of temporal persistence could significantly improve the performance of rainfall products in hydrological analysis.

Alternatively, we propose a simpler time series approach combined with the cascade schemes, which can be used with relative ease to introduce the temporal correlations observed in rainfall into spatial downscaling schemes. We interpret the scaling of rainfall in space as the combined effect of a spatial multiplicative cascade process based on large-scale forcing and a number of time series processes that relate rainfall intensity at a given cascade level to that of the past time steps or the history. Thus, the model is capable of mimicking the temporal persistence at all spatial scales involved in the problem at hand. We use a number of Markov processes (Haan, 1977) to model the autocorrelations.

The ability of a cascade scheme to treat the zero rainfalls explicitly (e.g. Over & Gupta, 1996) may be of particular interest to the run off analyst. In addition to the intensity, data analyses shows that zero rainfalls at various spatial scales, also have a strong temporal correlations. In the present model, zero rainfall persistence is modelled based on a simple conditional probability scheme.

After developing the theoretical framework for the proposal based on the features of spatial rainfall, the model's ability to mimic temporal persistence is demonstrated using hourly radar-based spatial rainfall observations for the central part of Japan. The radar-based rainfall estimates calibrated by raingauges known as radar-AMeDAS data (Makihara, 1996) was used for the present study. A rain storm from 14 May 1997 12:00 h to 17 May 1997 12:00 h, covering an area of 128 × 128 pixels bounded by 39.65°N, 134.5°W, 33.3°S and 142.4375°E was selected to demonstrate the operation of the model.

CASCADE DOWNSCALING

Multiplicative cascade schemes were first used to describe turbulent energy relations between eddies of different sizes in fluid flows. During the last two decades, the theory has increasingly been used to describe and model rainfall process in space and time dimensions. While the quantity of interest of the former case is energy, that of the latter is the quantitative rainfall. Figure 1 gives a schematic representation of a multiplicative cascade scheme in two dimensions. The process starts with a uniform intensity in the process domain. At each step, a uniform field is subdivided into four equal parts and each is multiplied by a cascade weight *W* that is derived from a specific probability distribution. The 128 × 128 data space used in the present study involves seven cascade steps.

In the present research, the β-lognormal model proposed by Over & Gupta (1996) was used to model the scaling in spatial dimension. A key feature of this model is the explicit modelling of non-rainy areas in addition to the rainfall intensity. In this model it is assumed that the zero–nonzero partitioning and the intensity process are independent of each other, which is not completely realistic. Typical uses of this model

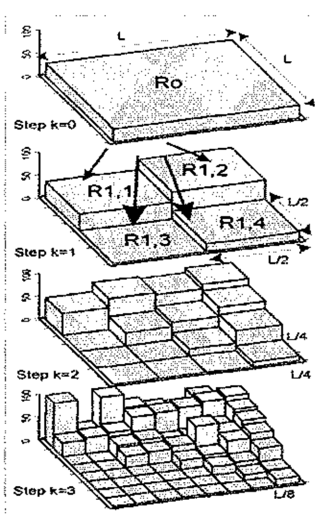

Fig. 1 A multiplicative cascade process in two-dimension. Uniform fields are divided into four and multiplied with the cascade weight *W* to obtain the next step.

for distributing a large-scale forcing amount into finer grids can be found in Jothityangkoon *et al.* (2000) and Pathirana & Herath (2002).

TEMPORAL CORRELATIONS

Physical understanding of the rainfall process, as well as numerous observations, confirms that it is one that evolves in space over time. In most cases storm movement in a specific direction is involved. However, these patterns are very much dependent on the atmospheric conditions, landforms, and the wind patterns involved in the specific location and time (for example, see Fig. 2). There were numerous attempts to improve the realistic representation of rainfall-related atmospheric phenomena using physically-based atmospheric models. However, in the context of a stochastic model these are hard to represent with an accuracy that justifies the increased complexity of model. The approach proposed in this paper is far simpler, with the assumption that the correlations of cascade weights can adequately capture the essential effects of storm evolution.

In order to understand the nature and magnitude of the correlations at different cascade levels, the following analysis was done: each spatial field of hourly series of rainfall snapshots was degenerated (the exact reverse of the process shown in Fig. 1) to obtain cascade weights W at each branch at each cascade level. The autocorrelations were calculated using the time series of weights corresponding to each pixel (i, j) at each cascade level k. Figure 3 illustrates these time series for a cascade process in one-dimensional space. A single value of autocorrelation for each cascade level was

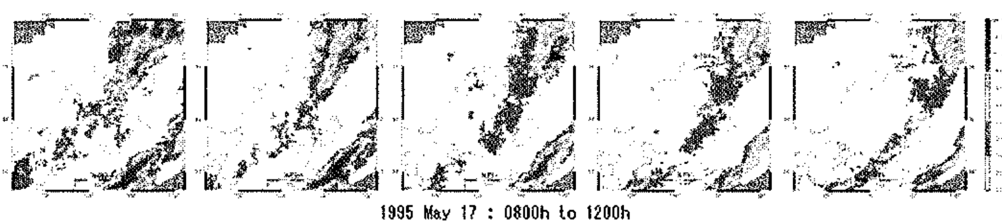

Fig. 2 A selected portion of a hourly spatial rainfall series of a storm over the central part of Japanese archipelago. Rainfall patterns show a clear evolution over time and a non-random directional movement.

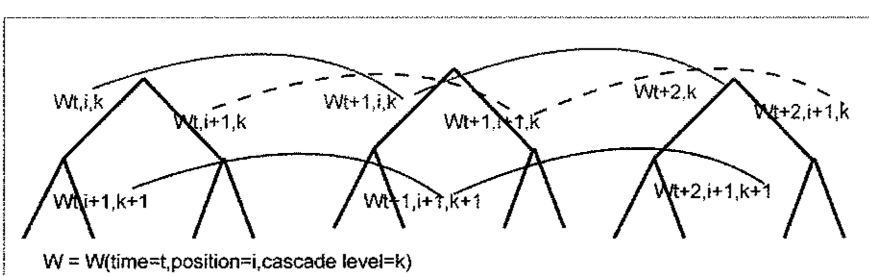

Fig. 3 The cascade weights at each cascade level are modelled as correlated processes. A one-dimensional cascade is shown for simplicity.

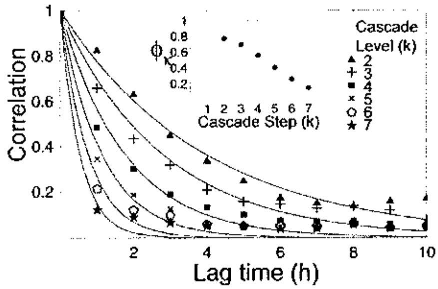

Fig. 4 Autocorrelations of cascade weights between adjacent rainfall fields. Fittings of a Markov process are also shown.

obtained by considering the time series for each pixel (i, j) for the same level k are realizations of the same time series process. In the case of the nth cascade step of a two-dimensional cascade, there would be 2^{2n} number of weights involved and thus, 2^{2n} number of correlation values will be averaged. The results of the analysis are shown in Fig. 4. The analysis shows that the cascade weights have significant autocorrelations and that they diminish in magnitude with the cascade step. Further, it is clear that a Markov process given by $W_t = \phi\ W_{t-1} + W$ can adequately represent the correlation process, where $0 < \phi < 1$, W is a random variable and t is time.

MODEL FORMULATION

The generator (the function which generates the weights W) of a typical β log-normal cascade model, adopting a notation slightly different from the original of Over & Gupta (1996), can be expressed in the following form:

$$P(W = 0) = 1 - e^{-\gamma};\ P(W = e^{\gamma-\sigma^2/2+\sigma X} = e^{G(\gamma-\sigma^2/2\sigma)}) = e^{-\gamma} \quad (1)$$

where X is a standard normal variable and $G(\mu,\sigma)$ indicates a normal distribution with mean μ and standard deviation σ. The model parameter γ is used to explicitly partition rainy and non-rainy areas and σ handles the intensity distributions by controlling the generator for non-zero W. Note that the cascade process is statistically conserved due to the fact that expected value, $E(W) = 1$. In order to introduce the temporal persistence to cascade modelling, this model should be modified to incorporate correlations among cascades adjacent in time. Since the cascade model requires the weights to be positive-definite, the logarithms of the weights, instead of cascade weights, were modelled. This does not cause a problem since the logarithmically transformed weights also show a good autocorrelation structure (Fig. 5). Then, with a Markov process as the basis for autocorrelations, the following generator can be proposed for the intensity formulation:

$$W_i = e^{\phi_k \log(W_{i-1}) + G(\mu_i',\sigma_i)} \quad (2)$$

where i is the time step, ϕ_k the Markov parameter for cascade level k and:

$$\mu'_i = \gamma_i - \sigma^2_i/2 - \phi_k(\gamma_{i-1}/2);\ \sigma'^2_i = \sigma^2_i - \phi^2_k\sigma^2_{i-1} \ \text{for}\ \ \sigma^2_i > \phi^2_k\sigma^2_{i-1} = 0\ \ o.w. \quad (3)$$

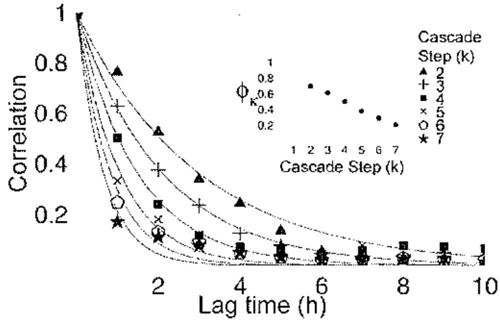

Fig. 5 Autocorrelations of logarithmically transformed cascade weights of observed rainfall.

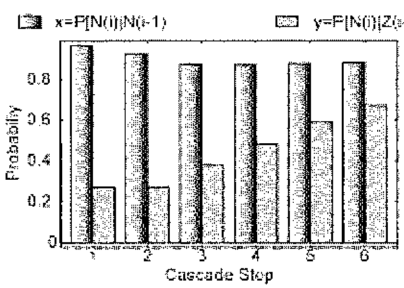

Fig. 6 Probability of rain in a previously rainy pixel $x = P_{N(i)} \,|\, P_{N(i-1)}$ and probability of rain in a previously dry pixel $y = P_{N(i)} \,|\, P_{Z(i-1)}$ at different cascade levels.

The last statements assume that $(G(\mu_i',\alpha_i')$ is independent of W_{i-1}. Note that the new generator is also conserved. In addition to the intensities, the rainy/non-rainy areas also have temporal correlations. Analysis of the conditional probabilities of rain occurrence at pixels that had/did not have rainfall in previous time steps showed that the probability of rainfall is significantly high if there was rainfall in previous step (Fig. 6). Thus, the conditional probability:

$$x = P[N(i) \,|\, N(i-1)] \qquad (4)$$

where i is the time step, N denotes non-zero rainfall, was used as a single value for the whole model to represent the persistence. Since there was no strong dependency of the quantity x with the cascade level (in contrast of ϕ) a constant value of x was used for all levels. For mathematical consistency, the model should make sure that the probability $y = P[N(i) \,|\, Z(i-1)]$ (where z indicates zero rainfall) should satisfy $0 \le y \le 1$. These details are not presented.

MODEL CALIBRATION

The calibration of the model should have two stages: the first stage is obtaining the parameters for the model given in equation (1). Since the typical use of the cascade model is to distribute a given uniform rainfall amount by a large-scale model among

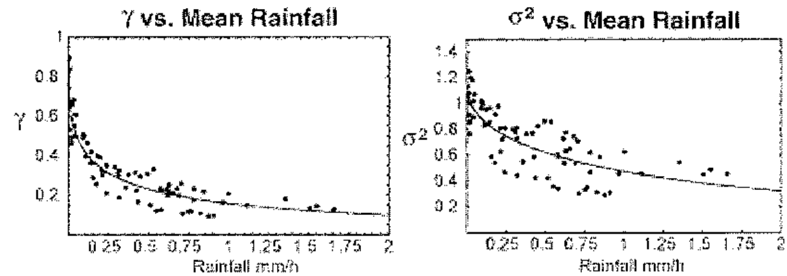

Fig. 7 Regression relationships of σ^2 and γ with large-scale forcing, obtained by data analysis.

the constituent pixels, these model parameters are expressed as regression functions of the large-scale forcing (Fig. 7). The second stage of calibration involves the determination of the temporal persistence parameters σ_k and x in equations (2) and (4). The former is found by fitting Markov processes to autocorrelation functions by least square fitting as shown in Fig. 5. The mean estimate of conditional probability given by data analysis (for all cascade levels) is used for the latter, x.

MODEL OPERATION AND RESULTS

Once a cascade model is calibrated, it can be used to spatially distribute a rainfall amount specified by large-scale forcing in a way that is statistically similar to observed data. Cascade models can accurately represent the spatial variability, intermittency and, sometimes, extra features such as zero value fractions and spatial heterogeneity (Jothityangkoon *et al.*, 2000; Pathirana & Herath, 2002). The additional feature of the proposed model is its ability to represent the temporal persistence that is present in observed data.

The procedure of downscaling a time series of large-scale forcing values using the model is as follows: First, values of model parameters σ^2 and γ are obtained from the regression relationships (Fig. 7). If the present time step is the first in the series, or if there was a zero rainfall amount in the previous step, the typical cascade downscaling based on weights obtained from equation (1) is used. If this is not the case, then equation (4) is used to divide the rainy fraction given by γ among previously rainy and previously non-rainy areas. Then equation (3) is used to determine weights for the non-dry portion of the cascade level. It should be noted that, according to the present model formulation, persistence of dry–wet partitioning x does not depend on the cascade level, though that of the intensity formulation ϕ_k does.

Figure 8 shows the autocorrelations found in simulated cascades. The Markov model could preserve the cascade level persistence comparable to those found by data analysis. Figure 9 shows two time series of spatial distributions, one downscaled using the typical cascade simulation without considering the autocorrelations and the other using the model proposed in this paper. The present model could introduce the temporal persistence similar to that found in observed data.

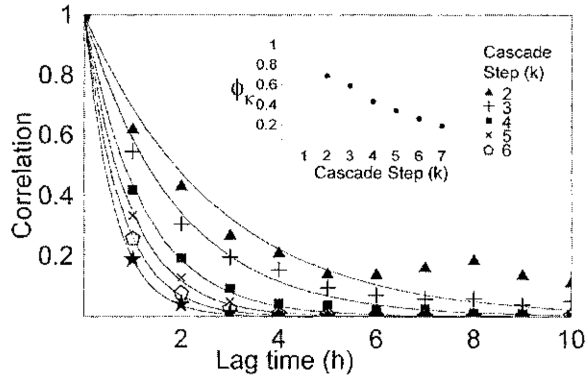

Fig. 8 Autocorrelations of logarithmically transformed cascade weights of simulated rainfall.

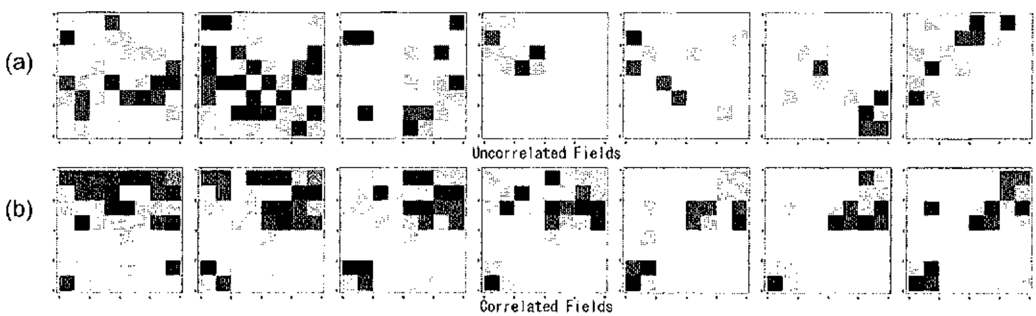

Fig. 9 Sample rainfall distributions using cascade theory: (a) typical cascade simulations, without considering temporal correlations, and (b) simulations using present model. Results produced at three cascade steps for clarity.

DISCUSSION

Spatial multiplicative cascade theory can be effectively used to distribute a rainfall amount prescribed by large-scale forcing on a spatial domain, into its constituent grids. However, when the cascade simulations of the large-scale forcing values on the temporal succession are done independently of each other, the resulting spatial distributions lack the temporal persistence that is shown by observed rainfall (Fig. 2). While these distributions faithfully reproduce the characteristics of variability and intermittency of spatial rainfall, they are not suitable as input for applications in surface hydrology for the above reason. The correlation technique proposed in the present paper modifies the cascade simulation scheme so that the pixel-level temporal persistence is introduced into cascade simulations using a set of simple time series models to determine cascade weights. The modified model could maintain in the simulated spatial rainfall fields and the temporal correlations that are present in observed spatial rainfall.

The temporal persistence in real spatial rainfall is maintained by physical mechanisms like wind such as caused cloud movement and rainstorm evolution over time. Since those mechanisms are not explicitly represented in the present model, it cannot realistically mimic the movement of rainstorms over time. The present method is significantly less complicated and easier to calibrate than the full three-dimensional treatment of rainfall. On the other hand, it is fully based on the empirical properties of the observed spatial rainfall and configurable to suit the rainfall patterns of a particular geographical location or season.

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