Weather Radar Information and Distributed Hydrological Modelling (Proceedings of symposium HS03 held during IUGG2003 at Sapporo, July 2003). IAHS Publ. no. 282, 2003.

Effect of radar beam geometry on radar rainfall estimation

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Abstract The frequency, accuracy and resolution of hydrological records is a major limitation in the accurate modelling of hydrological events. The weather radar provides real-time spatially continuous measurements covering a large area at short time intervals. However, considerable uncertainty remains in the procedures used to estimate the rainfall from weather radar observations. This uncertainty may be caused by the variability of raindrop size distribution, the variation of reflectivity with height and with range, and the temporal and spatial resolutions adopted for sampling the radar reflectivity. This paper accounts for the effect of the radar beam geometry as a function of distance. The conical shape of the radar beam causes the observed volume to increase with range from the radar, leading to both bias and an increase in the standard error associated with the measured reflectivity as a function of range. To remove the bias caused by the radar beam spreading, a simple-scaling transformation is proposed. The results show that the transformed reflectivity becomes relatively free from range dependent bias, which leads to more accurate relations with the measured ground rainfall than what is obtained otherwise. A 6-month long rainfall-reflectivity record from the Kurnell radar at Sydney, Australia is used to illustrate the efficiency and applicability of the reflectivity scaling transformation, compared to radar rainfall algorithms used conventionally.

Key words rainfall; scaling; weather radar

INTRODUCTION

Radar rainfall estimation traditionally involves the use of a parametric relation formulated based on measurements of radar reflectivity and ground (point) raingauge rainfall. However, such relations are often uncertain and their use in practical scenarios often leads to significant bias in the estimated rainfalls. Numerous factors are responsible for this uncertainty, including relating of reflectivity measurements that reflect above ground rainfall, to values of rainfall measured at ground level, use of point measurements of ground rainfall as a surrogate of pixel averaged values, the variability of rainfall drop size distribution, the variation of reflectivity with height and with range, the temporal and spatial resolutions adopted for sampling the radar reflectivity, and radar hardware miscalibration and noise. To accommodate for such uncertainties, a probabilistic approach such as the Probability Matching Method (PMM), (Rosenfeld *et al.*, 1993), has been used to eliminate the need for specifying a formal relationship between reflectivity and rainfall. The rationale behind the PMM method is that rainfall and reflectivity values for a specified exceedence probability can be considered to be equal to each other. It is assumed that the cumulative distribution function (CDF) is invariant with range.

A conical shape of the radar beam causes the volume of a radar bin to increase as the square of the distance to the radar. Therefore, the small intense features that are present in a rain field will be averaged out by the measurement process, thereby leading to an underestimation of the probability of high intensity echoes at far range. Assuming that reflectivity's CDF is independent of range results in an uncertain representation of the reflectivity, which leads to uncertainty in the radar estimated rainfall. Hence it is imperative that the measured reflectivity be transformed to a variable that can be considered to be free from range dependent bias before use in estimating radar rainfall. This paper presents two related concepts that attempt to address some of the problems identified above. Firstly, we formulate and evaluate a reflectivity scale transformation function that assumes reflectivity to be a simple-scaling variable; secondly we apply the scale transformation function in two radar rainfall estimation methods and evaluate the effectiveness of using the transformed reflectivity in estimating radar rainfall.

EFFECT OF RADAR BEAM SPREADING IN MEASURED REFLECTIVITY

The volume of a radar bin increases as the square of the range of the bin to the radar. The increased volume means that the small but intense features in a rain field are averaged out as the range from the radar increases. This means that the upper tail of the CDF will be biased at far range. This sensitivity of rainfall statistics to scale is well known and can be modelled using the scaling hypothesis for rainfall (Lovejoy & Schertzer, 1985).

Hourly raingauge rainfall obtained from 6-month data that occurred in Sydney, Australia, during November 2000 to April 2001 and the corresponding 1.5 km CAPPI radar reflectivity data were used to investigate the effect of radar beam spreading in the measured reflectivity. The effect of bright band and a different observation altitude at far range are the other sources of error that cause the range dependent bias in measured reflectivity. In order to avoid the biases caused by these two sources of error, only the measured reflectivity and raingauge data that lie within 100 km from the radar were used in this study. Note that the climatological freezing level of the Sydney area during the study months are above 2.5 km and the height of the base scan beam centre at 100 km from the radar is 1.9 km above the ground, which can be considered to be not overly different from 1.5 km. Therefore, we assume that there is no bias caused by the bright band effect and different observation altitude in the 1.5 km CAPPI data that lie within 100 km from the radar. To avoid the effect of noise and false interpretation caused by hail in the measured radar reflectivity, the reflectivity values that are less

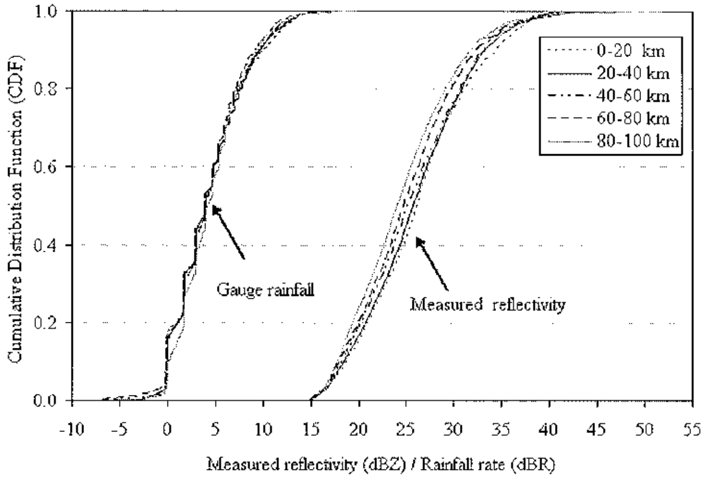

Fig. 1 Measured reflectivity and raingauge rainfall CDF.

than 15 dBZ and greater than 55 dBZ were excluded from the analysis. Raingauge rainfall data used in this study were obtained from tipping bucket raingauges. This type of raingauge records the time of bucket tips, hence they are subject to significant quantization error at low rainfall intensity. Hence, only the rainfall amounts that are greater than the volume of that gauge's tipping bucket were used in this study.

To show that the measured reflectivity CDF is a function of range, hourly raingauge rainfall obtained from the 6-month data and the corresponding 1.5 km CAPPI radar data were used to estimate the CDFs for raingauge rainfall and reflectivity data for each 20-km range interval, as illustrated in Fig. 1. In general, the probability of high reflectivity values decreases as function of range in the radar reflectivity, but this dependence is not observed in the gauge data and must therefore be an artefact.

SIMPLE SCALING HYPOTHESIS FOR HOURLY MEASURED REFLECTIVITY

From Fig. 1 we can see that the measured reflectivity can be considered as a random variable characterized by the same CDF with properly re-scaled parameters. A scale transformation function can be derived if one assumes that measured reflectivity at different ranges are connected through generalized scaling relation. Therefore, in this study the proposed transformation function is derived based on the simple scaling theory of rainfall. The hourly 1.5 km CAPPI data that lie within 100 km from the radar were used in estimating a scaling exponent. The simple scaling theory of time and space properties of rainfall proposed by Menabde *et al.* (1999) has been applied to the measured reflectivity. The reflectivity CDF of the 20-km range interval was selected as a reference reflectivity CDF. We assume that "simple scaling" holds for the measured reflectivity (in dBZ units). This assumption has been verified by estimating the scaling of moments of measured reflectivity at different moment orders (q). Figure 2 shows

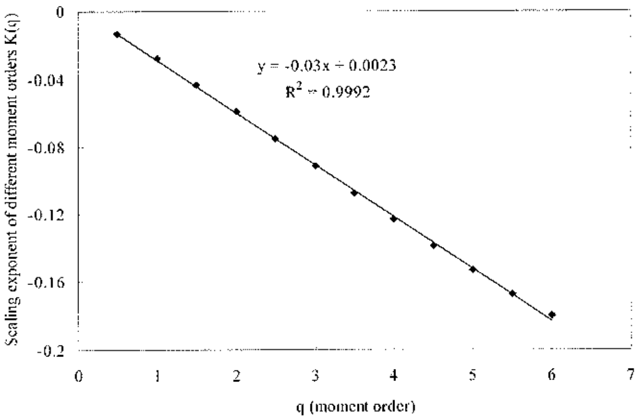

Fig. 2 Scaling exponent for hourly measured reflectivity.

that the dependence on q of the scaling of moments is precisely linear with scaling exponent of 0.03. This confirms the hypothesis about simple scaling in the measured reflectivity, therefore the proposed scale transformation function can be written as:

$$Z_{transformed(dBZ)} = \left(\frac{d}{20}\right)^{0.03} Z_{d(dBZ)} \tag{1}$$

where d (km) is the observation range of the measured reflectivity (beyond 20 km), Z_d (dBZ) is the measured reflectivity at range d. This proposed transformation function was used to transform the measured reflectivity at difference range interval to have same CDF as measured reflectivity at the 20-km range interval. The CDF of transformed reflectivity of each range interval was estimated as illustrated in Fig. 3. It can be seen that applying the scale transformation formula helps to transfer reflectivity's CDF of different range intervals to be close to the 20-km reflectivity's CDF. The mean of the transformed reflectivity of each range interval is corresponding to the mean of raingauge rainfall as shown in Fig. 4. This is a confirmation of the scaling hypothesis of the measured reflectivity, and thereby application of the scale transformation function helps to remove range dependent bias caused by the effect of the radar beam spreading. Radar and gauge data from three events that occurred across the Sydney area were then used to evaluate the effectiveness of using the transformed reflectivity in radar rainfall estimation.

APPLICATION TO KURNELL RADAR DATA

The Kurnell radar transmits radiation with wavelength of 5.3 cm and produces a beam with a 3 dB width of 0.94°. The reflectivity data are in Cartesian grids with 256 km × 256 km extent and a 1 km^2, 10-min resolution. Hourly reflectivity values were obtained by accumulating of 10-min reflectivity data. It should be noted that the hourly reflectivity values used in this study used the method of Fabry *et al.* (1994) to account for the movement of the rainfall field between the instantaneous rainfall

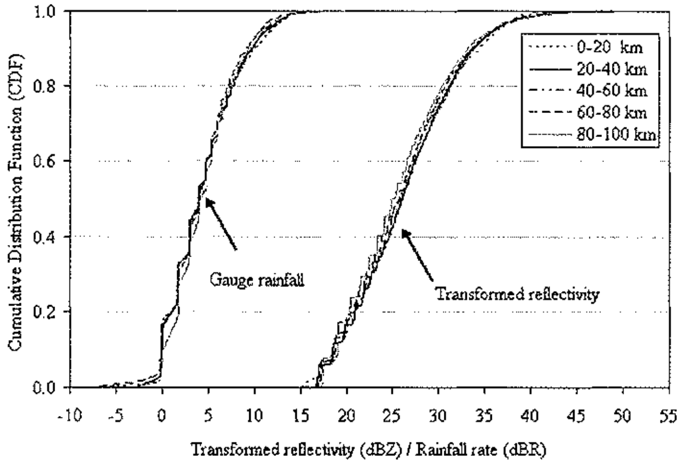

Fig. 3 Transformed reflectivity and raingauge rainfall CDF.

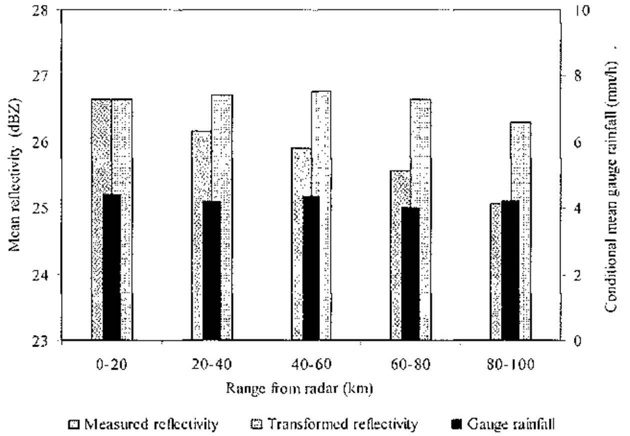

Fig. 4 Comparison of mean measured reflectivity and mean raingauge rainfall.

intensity fields produced by the radar. Raingauge data used in this study were obtained from a dense network of 222 hourly tipping-bucket gauge stations located within 100 km of the radar as shown in Fig. 5. Eighty-nine of these stations have a tipping bucket size of 1.0 mm. The other 133 stations have a tipping bucket size of 0.5 mm. Three case study events were used in the calibration and cross-validation. The first two case study events were widespread rainfalls that pass over Sydney during 30 January 2001–1 February 2001 and 19 April–22 April 2001, respectively. The third case study event was a convective event that occurred during 30 November–1 December 2000.

Altogether, four plausible rainfall calibration strategies were investigated. The methods were: (a) parametric Z–R relationship; (b) parametric Z–R relationship with transformed reflectivity; (c) PMM; and (d) PMM with transformed reflectivity. Note that for the parametric Z–R relationship, the parameter b was fixed to be equal to 1.5 and then calibrated parameter A by minimizing the mean square error (MSE) between

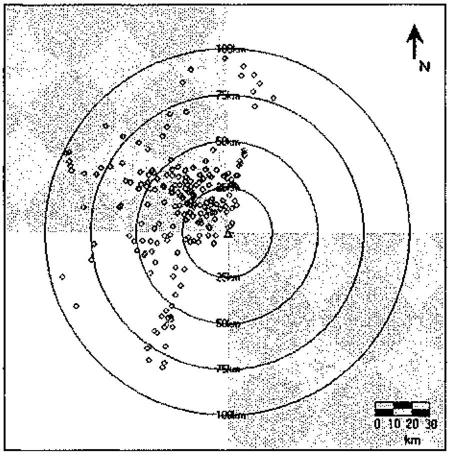

Fig. 5 Raingauge network.

radar and raingauge rainfall. The analysis was performed in an hourly time step by randomly selecting half of the available raingauges for the calibration; the remaining raingauges were used for the cross-validation of the algorithms. In order to investigate the range dependent bias induced by the effect of radar beam spreading, the gauge-radar (G/R) ratios at the raingauge's locations were estimated. The relative dispersion coefficient of variation about $\overline{G/R}$ (Wilson *et al.*, 1979) of each calibration and cross-validation set was calculated and used to evaluate the effectiveness in reduction of bias in the radar rainfall estimates after applying of scale transformation function in the measured reflectivity. The relative dispersion coefficient represents the ratio (expressed as a percentage) of the standard deviation of (G/R), where G and R represent the accumulated and the average (G/R) ratio over each storm. The accuracy of the radar estimated rainfall was investigated by a comparison of the R-square (R^2) statistic obtained from the calibration and cross-validation results. The R^2 is used as a dimensionless measure of model accuracy in the results presented.

RESULTS AND DISCUSSION

To investigate the sensitivity of the model performance obtained to the choice of raingauges network used in calibration and cross-validation, the analyses were repeated 100 times by changing the raingauge combinations used in the calibration and cross-validation. It is noted that the raingauge were selected in a purely random manner. The results presented in this study were the mean values obtained from the 100 re-calibrations. The results presented in Table 1 indicate that the use of transformed reflectivity in estimating radar rainfall can reduce the relative dispersion coefficient of variation about $\overline{G/R}$ approximately by 3% and 2% for calibration and cross-validation, respectively. It is interesting to note that the improvement in terms of relative dispersion is not obviously significant. This is because the random effects dominate, which causes the improvement in the bias to contribute little to the statistic.

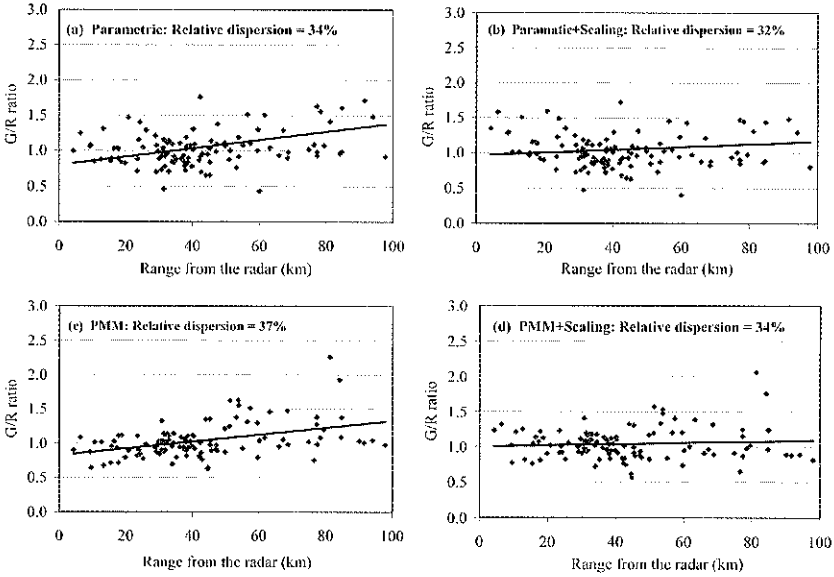

Fig. 6 *G*/*R* ratio for the validated raingauges of the 19 April 2001 event.

Table 1 Summary of gauge-radar comparisons of the calibration and cross-validation.

Events	Number of gauges	Storm duration (h)	$\overline{R}_g$ (mm)	Relative dispersion about $\overline{G/R}$ (%): Parametric	Parametric + Scaling	PMM	PMM +Scaling
30 Jan 01	105/104	70	6.2/5.9	34/32	31/30	35/33	32/31
19 Apr 01	111/111	82	3.2/3.2	29/28	27/26	31/30	28/28
30 Nov 00	108/107	47	3.5/3.7	45/45	42/44	44/45	41/44

$\overline{R}_g$ is the conditional mean of raingauge rainfall (R_g> raingauge's resolution mm h^{-1}).
(xxx/yyy) = (calibration /cross-validation).

However, it is apparent that slopes of the *G*/*R* ratios as a function of range are significant flatter when the transformed reflectivity values have been used in both the conventional radar rainfall estimation methods as illustrated in Fig. 6.

The accuracy of radar rainfall was estimated in term of the classical R^2 measures based on 100 sets of gauge combinations. These results are presented in Table 2. The difference in the R^2 results between the calibration and cross-validation of the two widespread events obtained from the parametric method are insignificant, which indicates the stability of the parameters when applied to the widespread rainfall. This optimistic assessment might be influenced by the fact that the validation gauges are placed in the vicinity of the calibration gauges, and thus are not fully independent. However, a reduction in accuracy of radar rainfall when validating for the convective case is evident in Table 2. This is because differences in rainfall drop size distribution are expected to be highly variable in both space and time in the convective rain, leading to a relatively greater level of independence between the calibration and cross validation gauges. The R^2 results of the three case study events indicate that an

Table 2. Sensivity in model performance.

Statistic	R^2				A-parameter	
	Parametric	Parametric + Scaling	PMM	PMM + Scaling	Parametric	Parametric + Scaling
30 Jan 01	0.699/0.677	0.703/0.687	0.684/0.671	0.689/0.678	60.11	63.53
19 Apr 01	0.443/0.441	0.450/0.448	0.325/0.211	0.333/0.225	97.99	111.93
30 Nov 00	0.257/0.184	0.260/0.184	0.022/-0.088	0.036/-0.08	275.32	305.44

(xxx/yyy) = (calibration /cross-validation)

increase in accuracy of radar rainfall is insignificant, even if the transformed reflectivity values that can be considered to be independent of range have been used in estimating radar rainfall. This might be because some other sources that cause uncertainty in radar rainfall, such as the variation of rainfall drop size distribution within the event, the reliability of measured reflectivity which is reduced with range due to observation problems, attenuation effect and the error of using point raingauge rainfall in representing mean-areal rainfall of a radar grid size, still have not been taken into account in this study. Although the improvement of the accuracy of radar rainfall estimates in the term of R^2 is insignificant, it still points to the effectiveness of applying scale transformation in the measured reflectivity.

Acknowledgements The authors gratefully acknowledge Mahanakorn University of Technology (Thailand) for funding the first author's PhD studies at the University of New South Wales. We also thank the Australian Bureau of Meteorology and the Sydney Water Corporation for providing the radar and raingauge data used in this study.

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