

## Evolution of a common variogram of water levels for monsoon and non-monsoon periods in an aquifer in a semiarid region

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**Abstract** Water levels were measured from 32 wells, fairly distributed in a watershed of a 60-km<sup>2</sup> area in a hard rock aquifer of a semiarid region of India. Data were collected once a month for 10 months between January 2000 and January 2001. The universal Kriging technique of the theory of regionalized variables with a linear drift was applied to analyse geostatistically the available water levels for each month. The bounded variogram, i.e. without the effect of drift for each set of data, was calculated with the help of a directional variogram in the perpendicular direction of the major flow. All the 10 variograms were analysed and then cross-validated to get a final acceptable model for each month. A common variogram was then evolved by averaging the nugget, sill and range values of monthly variograms corresponding to the monsoon period (July–October). Another common variogram was also evolved from the monthly ones of the non-monsoon period. It was then verified through a cross-validation test that in most cases these two common variograms were able to reproduce field values more satisfactorily than the individual monthly variogram.

**Key words** common variogram; cross-validation test; geostatistics; hard rock; monsoon; rainfall recharge; universal kriging; variogram; water level

### INTRODUCTION

Although, water level is a smooth varying parameter and also maintains continuity, it often shows high variability, depending on the variation in the system parameters arising from the heterogeneity of the formation as well as the stress applied to it such as rainfall recharge and extraction. Geostatistical techniques, using the theory of regionalized variables, have been applied to study water level in aquifers by a number of workers (Mizell, 1980; Dagan, 1985; Rouhani, 1988; Dong *et al.*, 1990; LaVenue & Pickens, 1992; Ahmed & Marsily, 1993; Ahmed, 1995; Roth, 1995, etc.). Rouhani (1990) has attempted a multivariate geostatistical model jointly using the time and space variability of water level, but was limited to theoretical developments. As water level is time varying and is often monitored using the same network depending on the required monitoring frequency, estimating water levels for all time periods following complicated steps of geostatistical estimation becomes cumbersome. Also the time variability of water level is influenced by external stresses that are often periodic in nature (both pumping and recharge due to monsoon rainfall). It is therefore possible to group water levels for certain time periods having similar behaviour and analyse them geostatistically for spatial variability.

The present study is based on the use of monthly water level data from a small drainage basin in an aquifer in a semiarid region in India. They have been analysed geostatistically and an attempt was made to evolve common variogram(s) for the two contrasting periods of the year, affected or not by the monsoon. The results of using these common variograms were compared to the monthly ones.

## STUDY AREA AND THE AVAILABLE DATA

The studied aquifer consists of hard crystalline rocks covering an area of 60 km<sup>2</sup>, between latitudes 17°6' to 17°11'N and longitudes 78°24' to 78°29'E where the top layer varies in thickness and weathering. Hence, a two-tier coupled system of weathered and fractured aquifers, exists over almost the entire area. Groundwater flow in the basin is mainly controlled by a coupled system of weathered and fractured rocks. Due to over-exploitation, the groundwater levels have declined and presently groundwater flow is mainly in the fractured rock aquifer under semi-confined to unconfined conditions. The area belongs to semiarid regions, subtropical climatic conditions prevail with minimum and maximum temperature of 22°C and 44°C respectively. The area receives more than 80% of its rainfall from the SW monsoon with an average normal annual rainfall of 812 mm. Ninety percent of the exploited groundwater is withdrawn through borewell structures. About 600 borewells exist in the area for irrigation purpose ranging from a depth of 30 m to 60 m fitted with submersible pumps. Water levels have been collected once a month for 10 months from 32

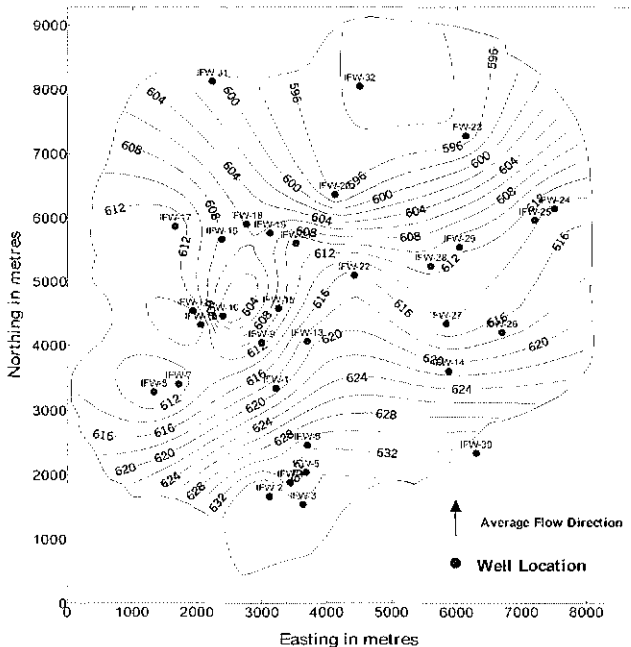


Fig. 1 Water-level contour map, January 2000 (Surfer without kriging).

wells fairly distributed across the area (Figs 1 and 2). The minimum and maximum water levels in the area during the year varies from 591 m (a.m.s.l.) to 646 m (a.m.s.l.) with an average variance of  $136.9 \text{ m}^2$ . The general flow direction of groundwater is from south to north (Fig. 1) and mostly follows the topography.

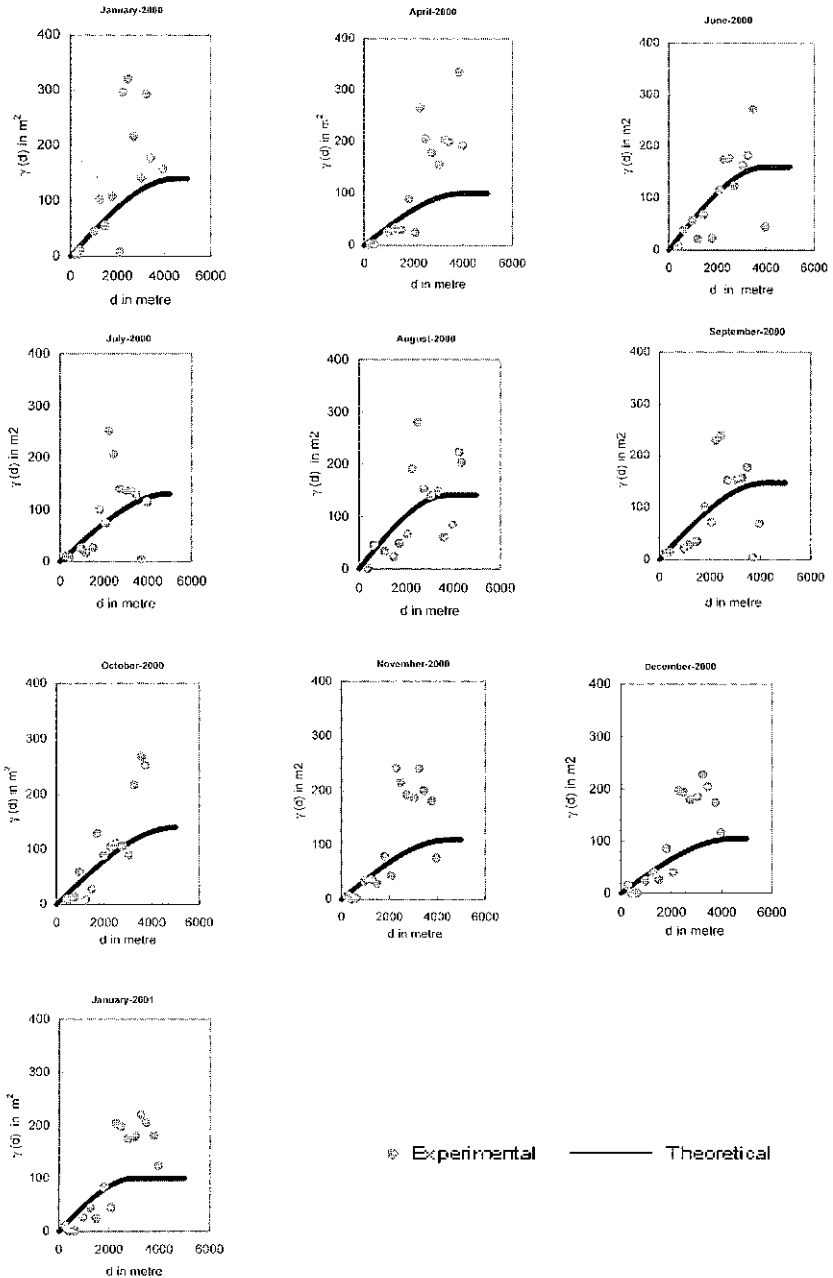


Fig. 2 Experimental and theoretical variograms for all periods.

## **GEOSTATISTICAL ANALYSIS**

Details on the theory of regionalized variables applied to groundwater hydrology are well documented by Marsily (1986), Issaks & Srivastava (1989) and Kitanidis (1997). The text below directly presents the work carried out for this study.

## **VARIOGRAPHIC ANALYSIS**

The presence of drift in a parameter produces an unbounded variogram and it becomes difficult to model it with a finite sill and range. Therefore, drift has to be removed directly or indirectly to obtain a bounded variogram. Initially unbounded variograms were obtained due to the presence of a clear drift in water levels (Fig. 1). Since a typical water level map, as shown in Fig. 1, depicts a single direction of major flow, it was decided to calculate directional variograms. Since the major drift terms are present in the flow direction, and they have practically no contribution in the perpendicular direction, calculation of an experimental variogram in the perpendicular direction of major flow provides a bounded variogram that can be assumed as the variogram of residuals. Calculation of experimental variograms for all the 10 months under consideration was performed in a direction perpendicular to the main flow direction and yielded bounded variograms. These variograms were calculated and modelled with theoretical models using an interactive program developed at the National Geophysical Research Institute (Ahmed, 2001). A theoretical variogram is fitted graphically by a visual trial and error method. The sill ranges from 160 m<sup>2</sup> (June 2000) to 100 m<sup>2</sup> (April 2000 and January 2001). The range of these variograms varies from 5000 m (July 2000) to 3000 (January 2001).

## **CROSS-VALIDATION TEST**

Due to the number of approximations made during calculation and modelling of the experimental variograms, the fitted models are bound to have ambiguities. Therefore, it was necessary to validate these variograms. A cross validation test was performed for all the months starting from January 2000 to January 2001 (Ahmed & Gupta, 1989) and revised variograms for each time period were then obtained (Table 1).

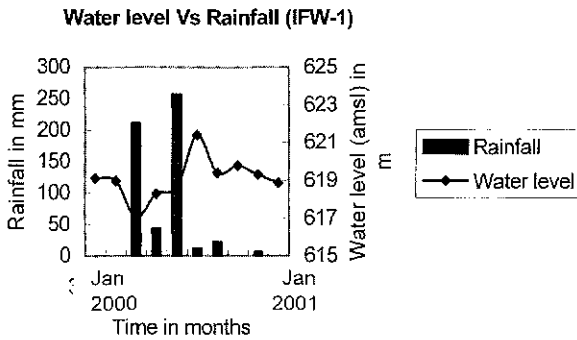
## **EVOLUTION OF COMMON VARIOGRAM**

To avoid calculation of experimental and theoretical variograms for each time period, an attempt was made to determine two common variograms for periods of several months. It was observed that during the monsoon-affected periods the spatial variability of water levels are different to the spatial variability of water levels in the remaining periods of the year (Fig. 3). Thus a common variogram was evolved by averaging the nugget, sill and range for the monsoon affected months (called monsoon variogram) and in a similar way another common variogram was evolved for the non-monsoon period (called non-monsoon variogram). It is important to note that monsoon periods for the rainfall in general are from July to October but for computing of the

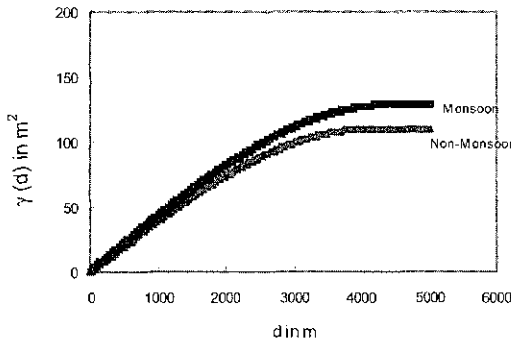
**Table 1** Comparison of cross-validation with common and individual variograms.

Month	Variogram obtained after cross-validation test				Month-wise		Non-monsoon $\gamma(d) = 0.0 + 110\text{sph}(4100)$		Monsoon $\gamma(d) = 0.0 + 128.4\text{sph}(4390)$	
	Model type	Nugget ( $\text{m}^2$ )	Sill ( $\text{m}^2$ )	Range (m)	SME*	SMRE	SME	SMRE	SME	SMRE
01/2000	Spheric	0	140	4500	38.45	1.03	38.37	1.20		
04/2000	Spheric	0	100	4000	24.78	0.69	24.99	0.65		
06/2000	Spheric	0	160	4000	63.49	1.02	64.74	1.54		
07/2000	Spheric	0	130	5000	35.75	0.97	36.13	0.93		
08/2000	Spheric	0	140	3750	44.67	0.72			47.93	0.99
09/2000	Spheric	0	147	4200	36.44	0.80			36.36	0.96
10/2000	Spheric	0	140	5000	29.62	0.84			29.91	0.80
11/2000	Spheric	0	110	4500	29.32	0.83			29.27	0.69
12/2000	Spheric	0	105	4500	27.91	0.86			27.80	0.68
01/2001	Spheric	0	100	3000	33.50	0.66	28.87	0.75		

\*where SME and SMRE are square mean error and square mean reduced error, respectively.



**Fig. 3** Variation of water levels of well IFW1 with rainfall in the study area.



**Fig. 4** Various types of common variograms studied.

variograms we considered August to December as during these months the water levels are influenced by the monsoon rains. The criteria of the limit between these two periods is that the rainfall recharge term is added during the monsoon period and it is absent during the remaining months. The common variograms (Fig. 4) were used for

performing a cross-validation test with the observed value from all the months, and the results have been compared with those of the individual months (Table 1).

## CONCLUSIONS

The present study shows that when performing geostatistical analysis of a parameter for different time periods, it is not absolutely necessary to carry out variographic analysis separately for all the time periods, that at times are quite cumbersome and ambiguous.

Although it is not always possible to evolve a single unique variogram for all time periods, this study with the help of cross-validation test has shown that if the yearly cycle is divided into two parts: monsoon and non-monsoon periods, it is possible to evolve a common variogram for each part. The common variograms during the validation test could satisfactorily reproduce the measured values of water level without losing much on the outcome (mean estimation error and mean reduced error) as compared to cross-validation tests using individual mensual variograms. This was possible for water levels as both the input (rainfall recharge) and the output (groundwater withdrawal for irrigation) more or less follow a cyclic pattern each year. Even in the case of low or high rainfall years, the special variability of water levels could be assumed to remain same.

The evolution of a common variogram helps the analysing of water levels for all the time periods that could be used, for e.g. calibration of an aquifer model etc. The study also helps in estimating water levels at any time period when all the wells could not be monitored due to any reason. This would have otherwise been extremely difficult because the variographic analysis in the absence of sufficient measurement becomes much more ambiguous.

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