# A completely-formed distributed rainfall–runoff model for the catchment scale

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**Abstract** A distributed model is proposed by integrating assimilation of input forcing onto grids, simulation of rainfall–evaporation–runoff processes at grids, computation of river routing, and calibration of spatial parameters. Spatial rainfall is assimilated with a spline interpolation, grid evapotranspiration and runoff are estimated with a soil–vegetation–atmosphere transfer scheme, river routing is calculated by a linear pool method, and parameters are calibrated by considering spatial heterogeneity. This daily runoff model is validated in the Hiji River catchment, Shikoku, Japan, resulting in good reproduction of daily discharges and annual water budgets.

**Key words** distributed model; rainfall; spline interpolation; interception; Hiji River, Japan; spatial parameter; river routing; runoff

## INTRODUCTION

A completely-formed distributed model is a model that has considered heterogeneity of all the four aspects: input forcing, grid processes, river flow routing, and parameters. Each of these aspects should be important to a complete distributed model. In this paper, such a completely-formed type of model is proposed. It is made up of assimilating input data onto grids, modelling rainfall–evaporation–runoff processes at each grid, computing river routing process, and calibrating parameters onto grids.

## CATCHMENT AND ASSIMILATION OF INPUT DATA

The Hiji River is located on the western Shikoku Island of Japan (Fig. 1), with a drainage area of 1009 km$^2$ measured at the Ozu discharge station. It is typical of forested mountains: more than 85% of its area is covered by conifer, broadleaf forest and fruit trees. The catchment is wide in the northeast–southwest direction but comparatively narrow in the southeast–northwest direction.

Observation gauges and stations are shown in Fig. 2(a). Available data of 27 years (1969–1995) include daily rainfalls at 21 raingauges in and around the catchment, daily mean temperatures at four weather stations, and daily mean discharges at four discharge stations. The catchment has an average annual precipitation of 1800 mm, average annual runoff of 1210 mm, and average temperature of 13.5°C.

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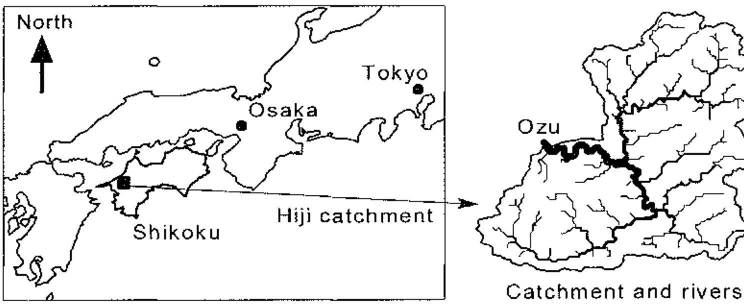


**Fig. 1** Location of the Hiji River and its catchment.

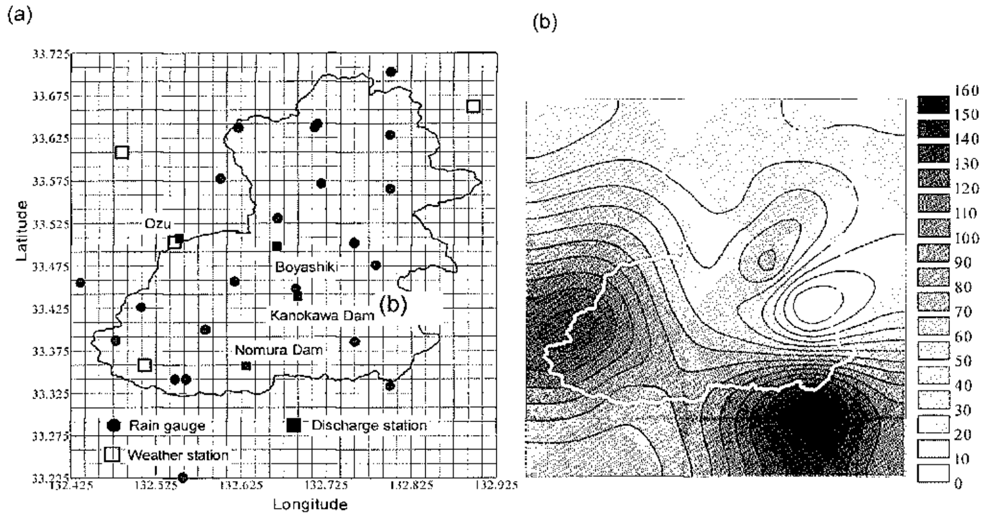


**Fig. 2** (a) Map showing observation gauges, and (b) assimilated rainfall distribution.

Two meteorological input variables, rainfall and temperature, have to be spatially distributed. A rectangular square of 2574 km$^2$ (latitude 33.225°–33.725°, longitude 132.425°–132.925°) is considered, covering the study catchment. This square is divided into 3600 small grids which have a side length of 845.6 m or an area of 0.715 km$^2$ (see Fig. 2). Spatial distributions of rainfall or temperature on each grid are derived using *in situ* data within this square.

Rainfall distribution is obtained by a spline interpolation method (Yao & Hashino, 2000), which uses the spline functions and creates grid rainfall from field data. Daily rainfall data at 21 gauges are utilized in the spline algorithm, creating rainfall values for each of 3600 grids. For example, the rainfall distribution on 25 July 1994 is illustrated in Fig. 2(b), with less rainfall in the northern and central parts and heavier rainfall in the southern and western parts.

Observed rainfall values at three other gauges (not used in the spline algorithm) are compared with the created results for 38 days in 1993, 1994 and 1995, giving a high correlation coefficient of 0.955 between the derived and observed values.

As for the distribution of temperature, a simple interpolation is applied. Temperature at a site is mainly determined by the altitude, if the latitude and longitude are given. In other words, spatial variation in a catchment like Hiji is mainly caused by the changes of altitude. A temperature decrease of 0.65°C per 100-m rise in the troposphere is widely accepted in meteorology, and will be used to derive spatial temperature distribution. Then daily temperature at any grid is estimated from the temperature at the Ozu observation station and the altitude difference between this grid and Ozu.

## MODEL DEVELOPMENT

A distributed model developed by the authors (Yao *et al.*, 1998; Yao & Terakawa, 1999; Yao & Hashino, 2000) is further improved and applied to the Hiji catchment. Simulations of the canopy interception process and river routing are improved. The Hiji catchment is divided into 5645 squared grids of 422.8 m width which is half of the grid size (845.6 m) previously used for input assimilation. Digital elevation data are used to create a river link network (Fig. 3).

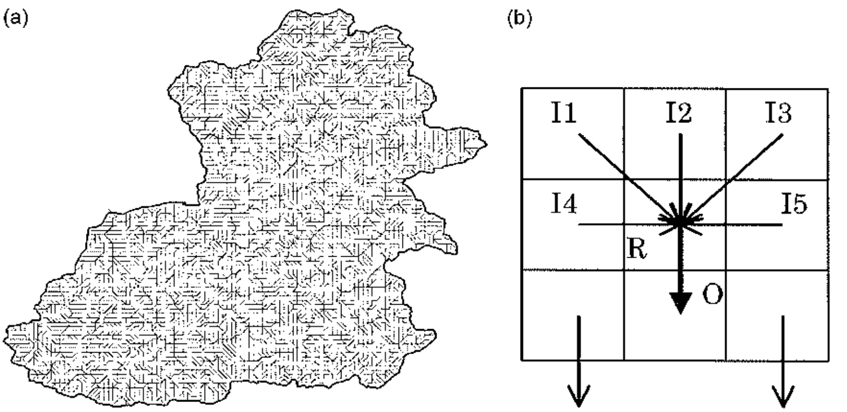


**Fig. 3** (a) River links of the catchment, and (b) example of river routing.

The water cycle processes in each grid are described with a grid sub-model. Rainfall interception in the canopy, evaporation from intercepted water or from soil, transpiration, soil water retention, and different flows on soil surface, in soils and groundwater aquifers are respectively estimated. These flows in a grid are supposed to enter the river link, forming the basic water source to river discharges. Then, water flow routing processes in the river network are calculated by a river routing sub-model. Integrated discharge at each link can be estimated.

### Grid sub-model

The grid sub-model is based on soil–vegetation–atmosphere transfer scheme, and is composed of four layers in the vertical: tree canopy, upper soil, lower soil and groundwater aquifer. Estimations are implemented day by day.

At first, daily potential evaporation, $E_P$ (mm day$^{-1}$) is estimated by the Hamon formula (Hamon, 1964) by using grid interpolated temperature. Evaporation, $E_C$, from intercepted rainwater on the canopy is expressed as (Hashino *et al.*, 1999):

$$\begin{aligned} &E_C = 0 && (W_{C0} + \alpha P = 0) \\ &E_C = W_{C0} + \alpha P && (0 < (W_{C0} + \alpha P) < \alpha r_c) \\ &E_C = \alpha r_c + Af(t) \cdot \alpha (P - r_c) \frac{E_P}{r_{c0}} && (\alpha r_c \le (W_{C0} + \alpha P)) \end{aligned} \tag{1}$$

where $P$ is the daily rainfall, $W_{C0}$ is the initial water content on canopy, α is the ratio of projected forest area to ground area, $r_c$ is the water-holding capacity of the canopy, which changes yearly, $r_{c0}$ is the $r_c$ value in 1969, $f(t)$ is the function of year $t$ reflecting the effect of leaf area change on interception, and $A$ is a coefficient.

Corresponding to equation (1), daily transpiration $T_R$ (including soil evaporation) is expressed as:

$$\begin{aligned} &T_R = BE_P \frac{r_c}{r_{c0}} \frac{W_{U0}}{W_{UC}} && (W_{C0} + \alpha P = 0) \\ &T_R = E_P - E_C && (0 < (W_{C0} + \alpha P) < \alpha r_c) \\ &T_R = 0 && (\alpha r_c \le (W_{C0} + \alpha P)) \end{aligned} \tag{2}$$

where $W_{U0}$ and $W_{UC}$ are the initial and field capacity water contents in upper soil layer, $B$ is a coefficient.

Outflows in upper soil are written as:

$$R_S = P_X + W_{U0} - T_R - W_{UM} \tag{3}$$

$$R_U = K_{UR}(W_{U0} - W_{UC}) \tag{4}$$

$$V_U = K_{UV}(W_{U0} - W_{UC}) \tag{5}$$

where $W_{UM}$, $K_{UR}$ and $K_{UV}$ are the saturation water content, saturated lateral flow coefficient and vertical flow coefficient respectively; $R_S$, $R_U$ and $V_U$ are surface flow, lateral flow and vertical flow in the upper soil, respectively. Water content at the ending time (e.g. initial water content for the next day) is written as:

$$W_U = W_{U0} - T_R + P_X - R_S - R_U - V_U \tag{6}$$

Similarly, outflows and water budget in the lower soil layer can be estimated.

Outflow, $R_G$, from the groundwater aquifer is expressed as:

$$R_G = \frac{K_G V_L + (2 - K_G) R'_G}{2} \tag{7}$$

where $K_G$ is the flow coefficient, $V_L$ is the vertical flow of the lower soil layer, and $R_G'$ is the ground flow on the previous day.

As a result, total runoff of a grid which will enter the river link is obtained as:

$$R = R_S + R_U + R_L + R_G \tag{8}$$

Estimations included in equations (1)–(8) are repeated for each day in a year, and daily runoffs of all grids are gained.

### River routing sub-model

As illustrated in Fig. 3, for one possible linkage, a river link receives inflows from neighbouring links and the runoff generated in its own grid, and gives its outflow. The routing process in this link is expressed with a water conservancy equation and a linear storage–flow relationship, just like a linear pool routing scheme.

$$\begin{aligned} \frac{\mathrm{d}S}{\mathrm{d}t} &= (I1+I2+I3+I4+I5+R)-O \\ O &= K_R \cdot S \end{aligned} \tag{9}$$

where $S$ is water storage in a link pool, $I1$–$I5$ are inflows from neighbouring links, $R$ is the runoff of the grid, $O$ is outflow discharge of the link, and $K_R$ is a flow coefficient dependent on links. Discharge of each link at any time is then calculated according to the flowing orders of links.

All the 5645 river links are divided in 144 levels from the most upstream link (with smallest level) to the river mouth link (with largest level), and the level of any link determines the flowing order for this link. Discharge routing is undertook from those links having smaller level onto the links having larger level.

## PARAMETER CALIBRATION

The distributed model has 14 important parameters, and they should be calibrated by considering their spatial variations. Five years (1969, 1971, 1979, 1993, 1994) have been chosen for the calibrations; calibrated results are listed in Table 1.

The water-holding capacity of the canopy, $r_c$, and the function of leaf area change $f(t)$ have different values for forested grids or other kind of grids, and they are changeable with year. They have smaller values in the 1960s, larger ones in the 1980s, because the forests grew and developed rapidly during the 1970s–1980s. The coefficients $A$ and $B$ have substantial effects on water components of intercepted rainfall and transpiration.

**Table 1** Parameters and calibrated results.

| Parameter | 1969 | 1971 | 1979 | 1993 | 1994 |
|---|---|---|---|---|---|
| $r_c$ (mm) | 1.0 | 1.0308 | 1.1539 | 1.0286 | 1.0143 |
| $f(t)$ | 1.0 | 1.0462 | 1.3846 | 1.2430 | 1.2214 |
| $A$ | | | 0.04422 | | |
| $B$ | | | 0.64538 | | |
| $W_{UC}$ (mm) | | 190.0–237.5 | | | 200.0–250.0 |
| $W_{UM}$ (mm) | | 225.6–282.0 | | | 240.0–300.0 |
| $K_{UV}$ (day$^{-1}$) | | 0.076–0.19 | | | 0.08–0.20 |
| $W_{LC}$ (mm) | | | 400.0–600.0 | | |
| $W_{LM}$ (mm) | | | 430.0–660.0 | | |
| $K_{UL}$ (day$^{-1}$) | | | 0.000–0.142 | | |
| $K_{LV}$ (day$^{-1}$) | | | 0.016–0.050 | | |
| $K_{LL}$ (day$^{-1}$) | | | 0.0000–0.0355 | | |
| $K_G$ (day$^{-1}$) | | | 0.025–0.050 | | |
| $K_R$ (day$^{-1}$) | | | 1.96–2.00 | | |

Parameters related to soil, aquifer and river channels are determined in two steps. First, a spatial distribution pattern is supposed, because deterministic or quantitative relations between a parameter and physical characteristics are not yet available in the research literature. Second, the changing range of a parameter within the catchment is optimized with hydrological data.

For instance, it is roughly supposed that the field capacity water content, $W_{UC}$, of the upper soil varies linearly with soil surface slope:

$$W_{UC} = W_{UC\min} + (L - L_{\max}) \frac{W_{UC\max} - W_{UC\min}}{L_{\min} - L_{\max}} \tag{10}$$

where $W_{UC}$ and $L$ are the field capacity and surface slope at any grid, $W_{UC\min}$ and $W_{UC\max}$ are the smallest and largest capacity values in all grids, $L_{\min}$ and $L_{\max}$ are the smallest and largest slopes in all grids. Then the smallest and largest capacity values are optimized by minimizing the errors in model estimation, and the results are 200 and 250 mm, respectively, for 1993. Similarly, the saturation water storage of the upper soil ($W_{UM}$), the field capacity ($W_{LC}$) and the saturation water storage ($W_{LM}$) of the lower soil layer are calibrated.

Saturated flow coefficients of the soil layers ($K_{UV}$, $K_{UL}$, $K_{LV}$, $K_{LL}$) are supposed to be linearly proportional to the altitude of the grids, having a larger value in higher

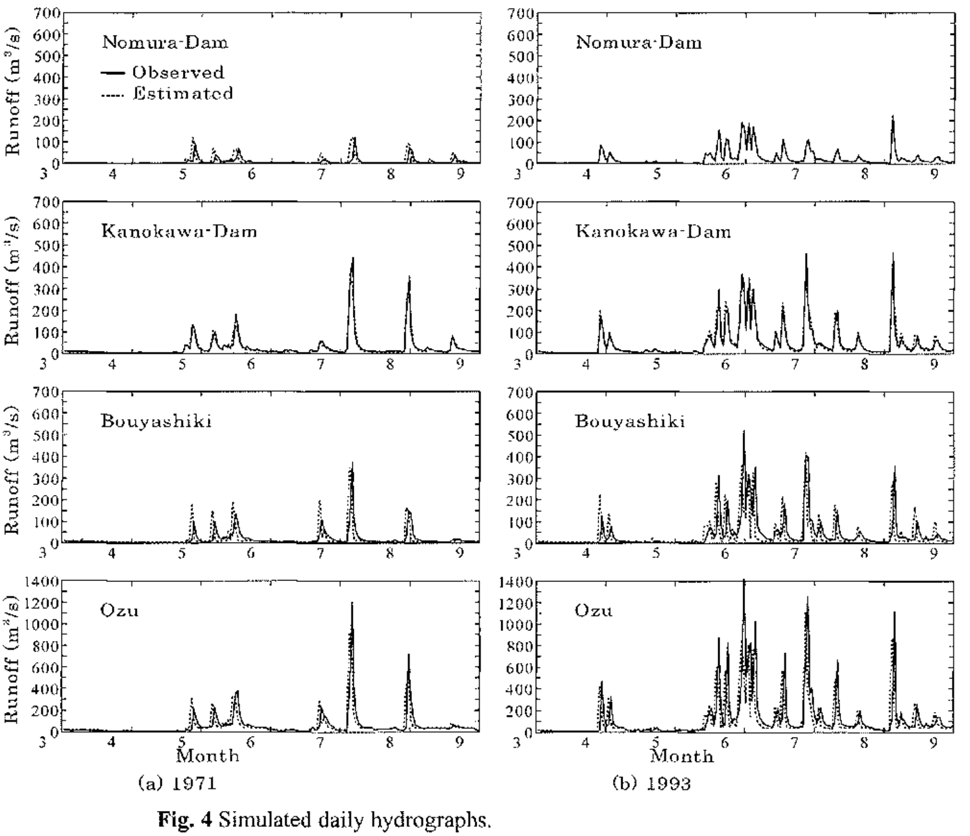


**Fig. 4** Simulated daily hydrographs.

**Table 2** Water budgets in 1971 for four drainage areas.

| Drainage area | Nomura Dam (168 km$^2$) | Kanokawa Dam (460 km$^2$) | Bouyashiki (371 km$^2$) | Ozu (1009 km$^2$) |
|---|---|---|---|---|
| Precipitation (mm) | 1574 | 1771 | 1614 | 1707 |
| Evapotranspiration (mm) | 634 | 656 | 633 | 650 |
| interception | 328 | 361 | 346 | 357 |
| transpiration | 306 | 295 | 287 | 293 |
| Runoff (mm) | 927 | 1164 | 1012 | 1096 |
| surface flow | 493 | 663 | 469 | 576 |
| upper soil flow | 79 | 102 | 119 | 111 |
| lower soil flow | 38 | 49 | 58 | 53 |
| ground flow | 315 | 349 | 367 | 356 |
| Observed runoff (mm) | 808 | 1224 | 944 | 1121 |
| Error (%) | 14.7 | –4.9 | 7.2 | –2.2 |

mountainous grids and a smaller value in lower plain grids:

$$K_{UV} = K_{UV\min} + (Z - Z_{\min})\frac{K_{UV\max} - K_{UV\min}}{Z_{\max} - Z_{\min}} \tag{11}$$

where $Z$ is the altitude of any grid, $Z_{\min}$ and $Z_{\max}$ are the smallest and largest elevations among all grids, $K_{UV\min}$ and $K_{UV\max}$ are the smallest and largest flow coefficient values which are optimized as 0.08 and 0.2 day$^{-1}$ respectively for 1993. Note that parameter values of the upper soil layer vary a little between the 1960s–1970s and 1980s–1990s.

The flow routing coefficient, $K_R$, is supposed to be proportional to the integrated drainage area of a river link. A link with larger drainage area usually has deeper water depth and is more effective in generating outflow:

$$K_R = K_{R\min} + (D - D_{\min})\frac{K_{R\max} - K_{R\min}}{D_{\max} - D_{\min}} \tag{12}$$

where $D$ is the drainage area of a link, $D_{\min}$ and $D_{\max}$ are the smallest and largest areas, $K_{R\min}$ and $K_{R\max}$ are the smallest and largest coefficient values. However, this parameter does not show a big spatial difference.

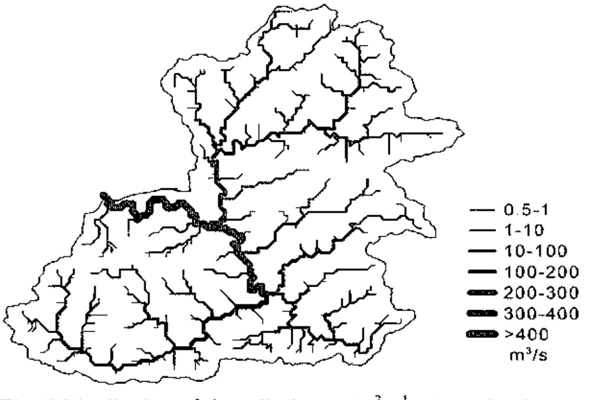


**Fig. 5** Distribution of river discharge (m$^3$ s$^{-1}$) along the river network.

Simulated discharge results of six months for 1971 and 1993 are illustrated in Fig. 4, using the parameter values in Table 1. Daily discharge hydrographs in a year are well reproduced at four points from the upper to the lower reaches. As for evaluation of the annual water budget, annual runoff estimations are close to the observed ones, as shown in Table 2 for 1971. Therefore, the proposed model proves applicable to this catchment.

Detailed distribution of discharge along the river network is provided, as shown in Fig. 5 for 4 July 1995. The thickness of link lines represents the discharge volumes.

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