Soil–Vegetation–Atmosphere Transfer Schemes and Large-Scale Hydrological Models (Proceedings of a symposium held during the Sixth IAHS Scientific Assembly at Maastricht, The Netherlands, July 2001). IAHS Publ. no. 270, 2001.

Parameterization of sub-grid effects in a large-scale hydrological model

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Abstract The methods of parameterization of sub-grid effects and their effectiveness are investigated using a distributed physically-based model of the hydrological cycle for two river basins situated in the steppe-forest zone of Russia. To estimate the sub-grid effects, it is assumed that these values are stochastic fields and two-parameter statistical distributions are used to describe variations of these characteristics inside the grid domain. Different assumptions on the change of the coefficient of spatial variation depending on the size of grid domain and the magnitude of characteristic under consideration were tested. It is shown that the dependence of the spatial variance of snow cover on the size of area can be obtained on the basis of the hypothesis of statistical self-similarity and the application of this dependence can essentially improve snowmelt runoff modelling. The rainfall, runoff and basin-averaged evapotranspiration are sensitive to the procedure of accounting for the sub-grid variation of saturated hydraulic conductivity and stomatal resistance, but the scaling of these parameters has comparatively small influence on the hydrographs.

Key words runoff; modelling; hydrological cycle; sub-grid effects; parameterization

INTRODUCTION

The present-day methods of numerical simulation of the hydrological cycle of a given area are based on splitting up this area into grid cells for which the model input and coefficients can be considered as spatially uniform. Depending on the problem under consideration, the size of grid cells may vary over a wide range; and at strong spatial variability of meteorological inputs and land surface characteristics, ignoring the sub-grid changes of these values, may considerably affect the hydrological cycle variables, even if we use very small grid domains. A theoretical framework for accounting of sub-grid processes of the hydrological cycle can be developed on the basis of areally averaging the main equations and using different assumptions on stochastic fields of the averaging inputs and model coefficients (for example, one such approach has been implemented by Kavvas *et al.* (1992) and Chen *et al.* (1994)). In general, this approach leads to a new complicated system of equations which needs additional information and new methods to be numerically solved. To simplify the problem, it may be justifiable to assume that the sub-grid variations of the meteorological inputs and model coefficients can be described by stable independent statistical distributions. However, the available experimental information is often only sufficient to calculate the parameters of these distributions for the area that is significantly larger than the grid cells or, *vice versa*, for a small part of considered area. Thus, it is necessary to assign the statistical parameters for domains without measurements, or to transfer these parameters from the larger or smaller domains (to make a spatial scaling of statistical parameters). In some cases, the statistical parameters in sub-grid areas can be found from the relationships between these parameters and the values that can be measured or determined on the basis of available measurements. For example, in Kuchment *et al.* (1986), for taking into account sub-grid effects in the generation of snowmelt runoff, the empirical relationships of the coefficient of variation of snow water equivalent and the depth of soil freezing with the means of these values were used; Kuchment *et al.* (1996) used an empirical relationship between the coefficient of variation of the saturated hydraulic conductivity and its mean value to describe the sub-grid effects in a rainfall–runoff generation model. More general approaches for assigning the sub-grid statistical parameters of input values and model coefficients can be developed on the basis of analysis of stochastic structure fields of these values for different spatial scales.

Let us represent a stochastic field $S(x)$ using a two-dimensional grid with coordinates ij as:

$$S_{ij} = M + a_k + \varepsilon_{ij} \tag{1}$$

where M is the mean value of $S(x)$ over the whole field, a_k is the deviation of the mean value over the kth mosaic region from M (mosaic variability), and ε_{ij} is a random component with a zero mean. If the field of the random variable $S(x)$ is uniform and isotropic in a broad sense, its mean value M and variance σ^2 will be the same for the whole area F and for any cell F_k. Therefore, statistical distributions dependent only on M and σ^2 will be identical for the whole field. However, it is known from experimental data analysis that spatial variations of geophysical characteristics commonly change with changing area size. Taking into account this effect, it is reasonable to assume that the statistical distributions of a given random variable $S(x)$ within any mosaic cell F_k will be the same as in the whole area F_k if a scaling transformation of this variable within F_k is made, e.g. the variable $S(x)$ is multiplied by a factor r^H in which r is a constant dependent on the ratio of F_k to F and H is a constant dependent on a measure of spatial correlation of $S(x)$. Such a property of a random variable is usually called statistical self-similarity. The condition of the equality of statistical distributions for F_k and F can be represented as the following relationship between all existing statistical moments of order n:

$$M[S^n{}_k] = r^{nH} M[S^n{}_F] \tag{2}$$

When the field of increments $I(h) = S(x + h) - S(x)$ is assumed to be uniform and isotropic, it is possible to construct the variogram:

$$\gamma(h) = M[S(x+h) - S(x)]^2 \tag{3}$$

If the variogram of the value of $S(x)$ has the power structure

$$\gamma(h) = \alpha h^{2H} \tag{4}$$

where α and H are constants, for the increments with steps of h and rh the following equality can be written:

$$I(h) = I(rh)/r^H \tag{5}$$

If we determine the statistical moments for both sides of equation (5), we have the condition (2) and, consequently, random variables having power structure variograms are statistically self-similar. By averaging $S(x)$ and variances for two embedded circles or squares with the centre in a point x_o and with areas F_k, F and taking into account equation (5), we obtain:

$$m_k - S_0 = r^H (M - S_0) \tag{6}$$

and

$$\sigma^2{}_k = r^{2H} \sigma^2{}_F \tag{7}$$

where m_k and $\sigma^2{}_k$ are the mean and variance of $S(x)$ over the area F_k, respectively; M and $\sigma^2{}_F$ are the same over the area F, and $r = \sqrt{F_k / F}$. If $S_0 = 0$, then, according to equations (6) and (7), the coefficient of variation CV of $S(x)$ within the mosaic regions F_k will be the same as for the whole area F (we used this assumption for taking into account the sub-grid variations of the saturated hydraulic conductivity in Kuchment *et al.*, 1986). The Brownian random process is an example of the self-similar random variable with the increments $I(h)$ being the Gaussian white noise and the variogram expressed by the function (4) at $H = 0.5$. In a more general model of a self-similar random process suggested by Mandelbrot (1982) and called a fractional Brownian process, the variogram is represented by the function (4) at $0 < H < 1$. If $H > 0.5$, the increments of this process are positively correlated and large-scale variations prevail; if $H < 0.5$, the increments are negatively correlated and small-scale variations prevail. As a measure of irregularity of a random surface and correlation of the large-scale and small-scale variations, Mandelbrot also introduced the fractal dimension $D = E + 1 + H$, where E is the topological dimension. Properties of geophysical random fields with a power structure function were used in many investigations in geostatistics (for example, Bruno & Raspa, 1989) and in application of fractal theory (for example, Burrough, 1983; Mandelbrot, 1982).

Thus, the problem of accounting for sub-grid variations of a spatial variable can be essentially simplified if it is possible to apply the hypothesis of statistical self-similarity. We have tested applying this hypothesis, using the distributed physically-based models of the hydrological cycle for two river basins situated in the steppe-forest zone of Russia.

Application of the hypothesis on statistical self-similarity of snow cover characteristics

To analyse the spatial variability of the snow cover, detailed snow surveys from the Sosna basin (a tributary of the Don; drainage area: 16 300 km^2) and the Nizhnedevitsk water balance station, located near the southern boundary of this basin, were used. On the basis of these measurements, the spatial variances of the maximum before-melt snow water equivalent, S_{max} for different area sizes were calculated; then the relationship between these ratios of the variances of S_{max} and the corresponding values of $r_a = \sqrt{F_k / F}$ was constructed on a logarithmic scale. As may be seen in Fig. 1, this relationship turned out to be close to linear and, consequently, the condition (equations

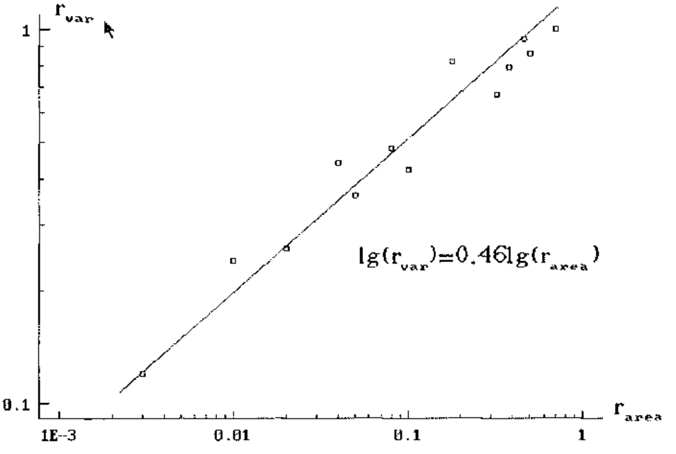

Fig. 1 The dependence of the ratio of the maximum snow equivalent variances on the ratio of areas for the Sosna basin.

(6) and (7)) for the spatial variances is fulfilled well enough ($2H = 0.46$; the fractal dimension $D = 2.77$). At the same time, the values of S_{max} averaged over different sizes of area also follow well the condition (equations (6) and (7)) for the means. In equation (2) the variogram of snow cover for an area of 100 000 km^2, which also includes the Sosna River basin, is given. This dependence was constructed on the basis of a special experimental snow survey by the State Hydrological Institute and is well approximated by a power function with exponent $2H = 0.56$ ($D = 2.72$).

To test the sensitivity of the snowmelt runoff generation model to different methods of accounting for sub-grid effects caused by small-scale variations of the snow cover, the simplified model of the Sosna River runoff generation, which is described in Kuchment & Gelfan (1996), was applied. The basin was represented by rectangular strips located along the river channels and on which a plane-parallel overland flow occurs. The kinematic wave equations are used to describe the overland and channel flow. The system of vertically averaged equations of snow pack formation is applied to describe the change of the snow pack properties and outflow from the snow pack. The snowmelt runoff losses are calculated by an empirical formula relating the rate of water absorption by the frozen soil to the saturated hydraulic conductivity of unfrozen soil and ice content in the frozen soil. The ice content of soil is calculated using a soil freezing model. The snowmelt rate is determined by the Kuzmin method. The model used contains three calibration parameters: the saturated hydraulic conductivity of unfrozen soil and roughness coefficients for overland and channel flow. The initial conditions in the equations of the dynamics of averaged snow depths and snow density are given by snow survey data from 30 equally spaced snow survey stations. The mean and variance of S_{max} for each area F_k were determined from equations (6) and (7) at $H = 0.23$. It is assumed that the value S_{max} has normal distribution in each grid domain. The area-averaged values of snowmelt rate outflow from snow pack, and soil absorption losses are calculated for each of the 30 grid domains. Three sets of calculations were carried out: (a) the mean value of S_{max} in the sub-grid area was assumed equal to the measured one and the variance σ_k^2 was set

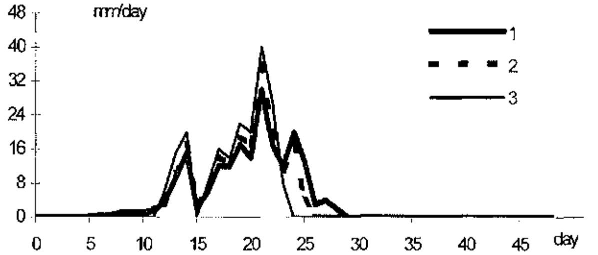

Fig. 2 Ouflow from snowpack averaged over the sub-grid area. (1: the same variation of S_{max} over sub-grid domain and over the whole watershed; 2: taking into account the self-similarity of the S_{max} fields; 3: zero variation of S_{max} in the sub-grid domain.)

equal to zero; (b) the mean value of S_{max} over this area was equal to the measured one, while the coefficient of variation was equal to the value calculated over the whole basin of the Sosna River for a given spring; and (c) the mean value and variance of S_{max} within the area were determined from equations (6) and (7). Figure 2 shows the outflow of snow-pack calculated for one of the 30 parts of the basin from the 1968 data for each set of calculations. It can be seen that accounting for the spatial non-uniformity of snow cover has led to the 5-day prolongation of the snowmelt period and to some decrease in the maximum rate of outflow from the snow-pack. Analogous results were obtained for other years. The decrease in the snow cover variance caused by the transfer from the whole basin to the grid area has led to a 3-day reduction of the snowmelt period.

Differences in hydrographs of the Sosna River calculated in the second and third set of calculations (5–7%) are less obvious than in the calculated outflow from the snow pack. This may be explained by smoothing due to large lag times of channel flow. Experiments were also performed to estimate the sensitivity of hydrographs to the measure of the statistical correlation of the field S_{max} expressed by the parameter *H*. The sensitivity of calculated hydrographs to the change of this parameter from 0.2 to 0.8 turned out generally to be low, but the differences in some hydrographs reached 10–12%.

Accounting for sub-grid variation of the river basin constants in a rainfall–runoff generation model

The applied model of the rainfall–runoff generation in the Seim River basin includes a description of overland runoff, vertical moisture transport in the "soil–vegetation–atmosphere" system, and channel flow in the river system. Both the overland flow and the channel flow are described by the kinematic wave equations, which are solved numerically using the finite element method. The Seim River basin was split into 433 finite elements. Depending on the domination of any soil type and land use, finite element areas were divided into four soil groups (typical chernozems, podzolic chernozems, serozems and flood plain-meadow soils) and 10 land-use groups. Most of the main soil constants (porosity, bulk density, maximum hygroscopicity, field capacity) and vegetation characteristics (leaf area index—*LAI*, plant height, root density, etc.) for each of these groups were assigned on the basis of measurements at the agrometeorological stations and observations during the KUREX experiments. Some model constants were calibrated using soil moisture measurements on plots with different soils and land use. Other constants were received from previous research in the neighbouring river basins and from the literature. The Manning roughness coefficients for overland and channel flow were calibrated against runoff hydrographs.

The estimation of sensitivity of the parameters of the model considered above showed that the most important parameters are the saturated hydraulic conductivity $K(0)$ and the minimum stomatal resistance r_{smin}. The influence of sub-grid random variations of the saturated hydraulic conductivity $K(0)$ on the runoff losses and hydrographs has been estimated in Kuchment *et al.* (1986, 1993, 1996), however, without taking into account changing the size of sub-grid area.

According to Di Federico & Neuman (1997), the hypothesis of statistical self-similarity can be applied to the spatial fields of $\log K(0)$. The investigations of the properties of random fields of r_{smin} are not known to us. In a series of our experiments associated with account of sub-grid variations of model parameters, it was assumed that the distributions of $K(0)$ and r_{smin} were gamma or lognormal inside each finite element with mean values equal to the relevant values of mosaic variation and with the same variances for the whole basin. This series has shown that random variation of the $K(0)$ inside finite element areas affect significantly the basin-averaged values of infiltration, runoff, soil moisture storage and soil moisture fluxes below the 1 m soil layer, but the influence of these variations of $K(0)$ on the basin-averaged evapotranspiration is small. At the same time, the random variation of r_{smin} inside finite element areas affects considerably the basin-averaged evapotranspiration and, to a certain extent, influences the runoff and the fluxes below the 1 m layer. A second series of numerical experiments was carried out to investigate the sensitivity of the basin-averaged components of the hydrological cycle to changes in the spatial variances of $K(0)$ and r_{smin} according to the self-similarity hypothesis (equations (6) and (7)). The grid domains with the same soil were combined and the mean values and the coefficients of variation of $K(0)$ and r_{smin} were determined for new sizes of area and the given values of H. Then the basin-averaged components of the hydrological cycle were calculated using 10, 86, and 433 parts of a basin and the lognormal distributions of $K(0)$ and r_{smin} inside these plots. A notable influence of area size change of $K(0)$ variance was revealed only for runoff. Effects caused by area size change of $K(0)$ variance gave a 10–15% change in the maximum discharge accuracy when changing the size of average grid area by approximately 50 times (for more details see Kuchment *et al.*, 1996). A change of H leads to only insignificant change of the hydrographs. A notable influence of area size change of variance was revealed only for r_{smin}. It is clear that these results are based on a very limited data set and need to be tested for a wider variety of climatic and geographical conditions.

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