Soil–Vegetation–Atmosphere Transfer Schemes and Large-Scale Hydrological Models (Proceedings of a symposium held during the Sixth IAHS Scientific Assembly at Maastricht, The Netherlands, July 2001). IAHS Publ. no. 270. 2001.

A physically-based rainfall–runoff model and distributed dynamic hybrid control inverse technique

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Abstract In this paper a physics-based rainfall–runoff model is developed utilizing an inverse technique based on hillslope hydrology, soil humidity and groundwater hydrodynamics, open-channel hydraulics theory and mathematical physical inverse problem theory. The model couples equations of two-dimensional transient subsurface flow, one-dimensional unsteady overland flow, one-dimensional groundwater flow, and one-dimensional channel flow. The inverse model allows for the spatio-temporal variation and parameters in hillslope, soil humidity, groundwater physical characteristics, and channel flow including vegetation interception, infiltration, evaporation, soil hydraulic conductivity, overland flow wave speed, subsurface flow wave speed, groundwater wave speed, stream wave speed and diffuse coefficient, etc. How to obtain the total unit-width discharge data from overground and underground flow to the channel becomes a key question in deciding whether or not to apply a physically-based mathematical model. A physically-based distributed hybrid control inverse model for rainfall–runoff is demonstrated by an application of a distributed and dynamic parameters inverse technique. The hybrid control technique (HCT) is proposed utilizing physics-based open-channel hydraulics and the mathematical physical inverse approach. The inverse solution provides a dynamic, distributed, nonlinear, and stable result. The model uses a finite difference method and a mathematical physical inverse method. The proposed HCT can represent the unit-width discharge process. The model and inverse technique have been applied to three sub-watersheds (of 52 200, 108 500 and 250 000 ha) within the Feng Man Reservoir catchment in China. There was generally good agreement between observed and simulated responses in all parts of the catchment , through the use of inverse parameters, upstream flow and total unit-width discharge. The results indicate that the inverse technique works quite well in flood flow simulation using 33 years of flood season hydrological records at the mouth of the channel.

Key words hybrid control technique; inverse technique; finite difference method; Feng Man Reservoir catchment; pulse spectrum method; physically-based rainfall–runoff model

INTRODUCTION

In recent years, hydrological models have been applied to solve various problems in water resources management, such as soil erosion, irrigation, water environment pollution etc. Among these, watershed models need to deal with spatial variability, scaling, and dynamic change. They need to consider explicitly linkages between hydrology, geochemistry, environmental biology, meteorology and climatology. Compared to the conceptual lumped models, the distributed models have a different structure. There are two kinds of structure used in distributed models. A loosely coupled distributed model mainly employs existing lumped conceptual models or methods for each cell or subsection of watershed. It does not, however, contain spatial structure. The other variant is the closely coupled distributed model that employs numerical schemes to establish the temporal and spatial relationships between adjacent grid cells or subsections of a watershed.

A physically-based mathematical inverse model has been developed to simulate the behaviour of rainfall–runoff of the watershed continuum. It expresses the interactions between the watershed subsystems by the kinematic wave equation of conflux. The model calculates the spatio-temporal variation and parameters in the watershed subsystems by mathematical inverse formulae that permit the simulation of many complex and physical processes. The model comprises four subsystems coupled together: (a) a lateral inflow and channel-head inflow counter-distributed hybrid control model; (b) a normal rainfall–runoff model; (c) a distributed and dynamic parameters inverse model; and (d) a real-time flood forecast model.

The model can inverse lateral inflow and channel-head inflow based on real flow measure information at the mouth of the channel. The temporal and spatial parameters and variables were calculated by the inverse model. A real-time flood forecast method is suggested by inverting the spatial and temporal parameters. These methods are strongly related to each other. Hence they are introduced together in this paper.

A COUNTER-DISTRIBUTED AND CONVERSE-BOUNDARY HYBRID CONTROL TECHNIQUE (HCT)

In theory, distributed rainfall–runoff models need to produce the spatial hydrological information such as overland flow , subsurface flow and groundwater flow. In practice, there is great difficulty in checking the spatial information produced, such as the lateral inflow and channel-head inflow. One of the main factors restricting the development of continental-scale hydrological models is the lack of net inflow observations, particularly total unit-width discharge data from overground and underground to the channel in medium-sized (50 000–100 000 ha) and large (>100 000 ha) watersheds. Hydrological stations often report discharge data only at the mouth of the channel. Currently, it is nearly impossible to get observed distributed information to calibrate the models. How to obtain the total unit-width discharge data from overground and underground to channel, becomes a key question in deciding whether or not to apply a physically-based mathematical model. The proposed HCT can provide the unit-width discharge process.

The problem of using a counter-distributed and converse-boundary hybrid control method to drive the one-dimensional convective diffusion equation of channelled runoff exactly to a given terminal condition (i.e. reported discharge data at the mouth of the channel) in a finite time T can be represented as follows. Adopting the case of two rectangular planes plus a channel forming a V shape (Wooding, 1965) leads to the following system of equations:

$$\frac{\partial Q}{\partial t}+c(y,t)\frac{\partial Q}{\partial y}-d(y,t)\frac{\partial^2 Q}{\partial y^2}=-d(y,t)\frac{\partial q_u}{\partial y}+c(y,t)q_u \qquad (1)$$

for $(y, t) \in Q = [0, l] \times [0, T]$, with initial condition:

$$Q(y, 0) = \psi_0 \qquad \text{for } y \in [0, l] \tag{2}$$

and given boundary condition:

$$Q(l, t) = Q_s(t) \qquad \text{for } t \in [0, T] \tag{3}$$

where ψ_0 represents initial record discharge data. The function $c(y, t)$ is wave speed, $d(y, t)$ is a diffuse coefficient, y is the distance along the river flow direction, t is time, and l is the length of the reach. The term $Q(l, t)$ represents the discharge process of distance y; $Q_s(t)$ is the recorded discharge data at the mouth of the channel.

If $I(t)$ is the channel-head inflow, and $q_u(t)$ is lateral inflow, i.e. if the total unit-width discharge data from overground and underground to channel are needed, it is required to determine two control functions, $I(t)$ and $q_u(t) \in L^2$ [0, T], using the boundary conditions:

$$Q(0, t) = I(t) \tag{4}$$

$$q_u(t) = I(t) - q_y \tag{5}$$

where q_y is the headstream water for $t \in [0, T]$ such that the solution of equations (1)–(5) at time t matches exactly a given function $I(t)$and $q_u(t) \in L^2$ [0, T]. In fact, two different types of control problem arise synchronously:

(a) *Boundary control* where $c(y, t)$ and $d(y, t)$ are two given functions, and it is required to determine $I(t)$ and $q_u(t)$; and
(b) *Distributed control* where $I(t)$ and $q_u(t)$ are given, and it is required to determine $c(y, t)$ and $d(y, t)$.

It is the purpose of this paper to describe a numerical method for calculating the required control functions in the case of upper boundary control and distributed control. When $I(t)$ has been given, only $q_u(t)$, $c(y, t)$ and $d(y, t)$ need be calculated. Then equation (4) becomes a boundary condition.

The boundary and time control problem of inverse rate of lateral inflow and channel-head inflow, subjected to a convection and diffusion boundary condition, is analysed by use of the finite difference method and regular method. In engineering and scientific computation it is often necessary to devise an optimal control of distributed parameter systems, which are characterized by one or several parameters distributed in space. There are several means of influencing the response of rainfall–runoff systems: one is distributed (distributed control), the other is by the boundary of the domain (boundary control), and the third is both (hybrid control). In recent years considerable attention has been directed towards inverse or system problems in linear and nonlinear structural dynamics. Inverse problems are grouped under four general classes: synthesis, system parameter identification, control, and construction identification.

The purpose of this model is to address an integrative method of inverse control problems belonging to the last two categories. Although it is possible to formulate the optimal control problem corresponding to many physical systems and to derive a set of optimal conditions, it is not easy to obtain the solution. After embedding the boundary control problem into the time control problems, the essential difficulty is that the values of the solutions are unstable by current optimal control methods. Any attempt at a numerical solution of a problem of exact controllability, based on a forward stepping difference procedure involving simultaneous discretization in space and time, will fail due to the complexity and instability of the resultant system of equations. The instability is due to the fact that a small change in a given control function will produce a large change in the solution of the control functions, whether boundary or distributive. In a forward stepping difference method with a control function, this inherent instability in the problem causes large changes in the controls for small changes in the numerical accuracy of the difference procedure. Consequently we discretize in space and time, using the result:

$$\text{(I)} \begin{cases} AQ = r \\ \text{when } Q(y,t) = Q_s(t), Q(0,t) = I(t) \end{cases} \tag{6}$$

$$\text{(II)} \begin{cases} AQ = r \\ \text{when } Q(y,0) = Q_s(0) \end{cases} \tag{7}$$

where Q is unknown, (I) $Q = [q_u, Q_2, \ldots, Q_{N\text{-}1}]^T$, (II) $Q = [I, Q_2, \ldots, Q_{N\text{-}1}]^T$, $q_{uj} = I_j - q_y$; A is the $n \times n$ tridiagonal matrix; r is the n vector, where the average values of c, d and q_y have to be estimated by optimization technique. If we now substitute c, d and q_y into equation (6) or (7), in the boundary control unknown variable $Q(y, t)$, $I(\ )$ and $q_u(y, t)$ can be determined by equation (6) or (7) using Tikhonov's regularization method (Tikhonov & Arsenin, 1977).

This system of difference equations (equation (1)) together with the initial condition (equation (2)) and the boundary condition (equation (4)) leads, by generalized pulse-spectrum technique, to:

$$\frac{\partial G}{\partial t} + c(y,t)\frac{\partial G}{\partial y} - \mathrm{d}(y,t)\frac{\partial^2 G}{\partial y^2} = \delta(y - y', t - t') \tag{8}$$

A type of the numerical solution is presented in accordance with the Fredholm integral equations of the first kind:

$$\begin{aligned} Q_{Nq}^S - Q_{Nq}^n = \sum_{i=1}^{N}\sum_{j=0}^{M-1}\Bigg\{ & \delta D_{ij+1}^n \left(G_{ijNq} - G_{i\ 1jNq}\right)\left(Q_{i-1j+1} - Q_{ij-q} + \Delta y q_{uij+1}\right)\frac{1}{\Delta y^2} + \\ & C_{ij+1}^n\left[\frac{Q_{ij+1}}{\Delta y}\left(G_{ijN1} - G_{i-1jNq}\right) + q_{uij+1}G_{i-1jNq}\right] + \delta q_{uij+1}^n\left[\frac{D_{ij+1}}{\Delta y}\left(G_{ijNq} - G_{i+jNq}\right) + C_{dij+1}G_{i-1jNq}\right]\Bigg\} \end{aligned} \tag{9}$$

where $\delta Q = Q_{Nq}^S - Q_{Nq}^n$ is an error of the recorded discharge and calculated discharge at the mouth of the channel. The values of δD_{ij}, δC_{ij}, δq_{uij} and δQ_{ij} can be obtained by using Tikhonov's regularization method for solving nonlinear operator equations which are ill-conditioned:

$$\begin{cases} D_{ij+1}^{n+1} = D_{ij+1}^n + \delta D_{ij+1}^n \\ C_{ij+1}^{n+1} = C_{ij+1}^n + \delta C_{ij+1}^n \\ q_{uij+1}^{n+1} = q_{uij+1} + \delta q_{uij+1}^n \\ Q_{ij+1}^{n+1} = Q_{ij+1}^n + \delta Q_{ij+1}^n \end{cases} \tag{10}$$

We have thus constructed distributed variables and the parameter control problem. The distributed value of the variables (i.e. $q_u(y, t)$, $Q(y, t)$) and parameters (i.e. $c(x, t)$, $d(x, t)$) can be computed using equations (1), (8), (9) and (10). The result of the above boundary control is obtained by way of the initial estimated values of the distributed variables and parameters.

The solution is sought by applying nonlinear numerical methods and stabilization inverse techniques. The controls resulting from this algorithm are likely to be most useful to the specific problem analysed in this paper: the inverse rate of lateral inflow and channel-head inflow. The application of the method to the rate of lateral inflow and channel-head inflow in the channel is demonstrated by applying it to three natural watersheds. Finally the results of these counter-control tests indicate that this counter-time and converse-boundary hybrid control algorithm is accurate and has good stability and convergence properties.

PHYSICS-BASED RAINFALL–RUNOFF NORMAL MODEL

A physically-based mathematical large-scale hydrological model for a large (>100 000 ha) and medium-sized (50 000–100 000 ha) catchment has been developed as a set of partial differential equations. The interactions between the watershed subsystems has been taken into consideration by mathematical formulae that permit the simulation of many complex and spatially-distributed and time variant physical processes. The model was based on the combination of the continuity and momentum. These ten partial differential equations were coupled to form unsteady overland flow, subsurface flow, regression recession flow and saturation overland flow, phreatic groundwater flow, and channel flow. The non-stability, nonlinear, nonhomogeneous, saturated and unsaturated flow modules use a finite difference method for numerical solution.

DISTRIBUTED AND DYNAMIC PARAMETER INVERSE MODULE

The inverse formulations can be derived directly from the hydrodynamic partial differential equations (Li, 1994, 1995). The inverse rate of lateral inflow has been presented in the above HCT model. In using a hydrodynamic model to calculate the production and collection of flow, the first requirement is to calibrate parameters and variables of the model (such as hydraulic conductivity, specific storage, wave velocity of overland flow, channel wave speed and diffusion coefficient, etc.) and identify state variables (such as the infiltration process, evaporation and transpiration processes, etc.) with the inverse model using the field recorded data. The derivation of land surface parameters and part state variables is solved as a the distributed calculation, nonlinear problem by the pulse spectrum technique (PST). It has been treated as an instability problem using the regularization method, and it has also dealt with the dynamic calculation problem by gliding the m groups data before t time . It was proved effective in determining the temporal and spatial parameters and state variables using basin outlet recorded channel flow data. The details of this model are not discussed further in this paper. More detailed discussion about the temporal charge and spatial distribution of hydrological parameters and state variables is presented in a separate paper.

REAL-TIME FLOOD FORECASTING MODEL

A physically-based real-time flood forecasting technology has been developed (see Li, 1998). The time-varying distributed parameters can be obtained by using the distributed and dynamic parameter inverse methods detailed above. When we use parameters in t time instead of in $t-\Delta t$, or $t-2\Delta t$, the real-time flood forecasting process can be carried out. In practical use, the methodology is to calibrate the time-varying distributed parameters against lateral inflow and channel outflow information based on field measurements or inverse boundary control. The validity of this approach has been to demonstrate the real-time flood forecasting process correctly for the Feng Man Reservoir catchment, northeastern China, and for its three sub-catchments, Mingli, Jiaohe and Hengdaozi. A variety of catchment responses can be forecast; the method can not only represent channel outflow, but also test various discharge components.

The three rivers Mingli, Jiaohe and Hengdaozi flow into the upper reaches of the Songhuajiang River at different places. The length of the Mingli, Jiaohe and Hengdaozi is 50, 63 and 40 km, respectively. The groundwater table depth is from 1.6 to 32 m. Average annual precipitation for the three areas of the valley is about 750 mm. The area of the Feng Mang Reservoir catchment is about 4 250 000 ha and that of the Mingli, Jiaohe and Hengdaozi sub-catchments is about 108 500, 250 000 and 52 200 ha, respectively. Flood data were sampled at 6-h intervals.

RESULTS AND DISCUSSION

The 33 years (between 1958 and 1995) flood season hydrological records at the mouth of the channel were chosen for simulation, representing a variety of rainfall events and catchment responses. Some of the test runs are illustrated in Figs 1–2. All simulations were made for the period from 1 June to 30 September for the flood season in each year.

The two hydrographs to be inverted were the lateral inflow (Fig. 1(a)) and channel-head inflow (Fig. 1(b)) information for the flood events in Mingli sub-catchment (1 June–30 September 1995). The inverse results of HCT, for instance the lateral inflow and channel-head inflow information, were reverted to the outflow equation (1) for recalculating outflow at the mouth of the channel. The lateral inflow results were a reasonable representation of net inflow from to the slopes field to channel. Some of the test runs are illustrated in Fig. 2, which shows the hydrographs equivalent to 5856 h of comparison of observed and calculated data (for 1 June–30 September 1983 and 1 June–30 September 1995).

Table 1 shows the results for which the lateral inflow and channel-head inflow from HCT revert to the outflow equation (1).

The distributed wave speed along flow direction is show in Fig. 3(a). Figure 3(b) describes a distributed diffusion coefficient for the Fengman Reservoir. The maximum values occurred at about 5.5 km from the reservoir dam wall. The resistance is minimal here, and the water depth is at a maximum, as are the distributing wave speed and diffusion coefficient. The temporal variations of wave speed and diffusion coefficient are large and the reasons remain unknown for the moment.

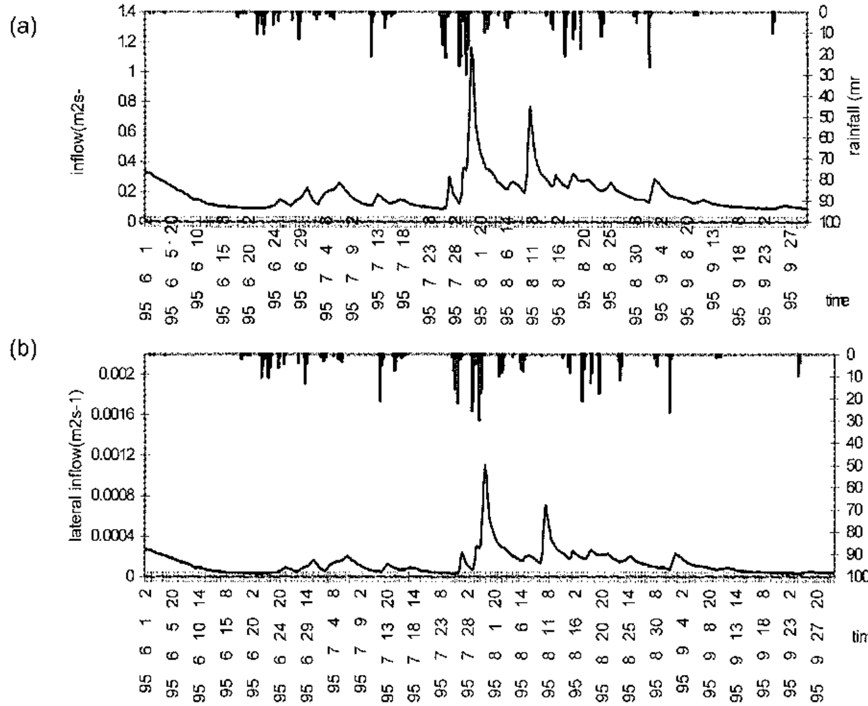

Fig. 1 Hengdaozi (a) inverse upper inflow processes, and (b) inverse lateral inflow processes (1995).

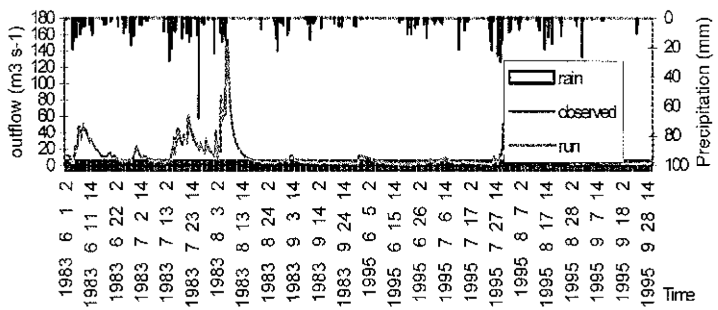

Fig. 2 Comparison of observed and calculated hydrographs, Hengdaozi (1983, 1995).

Table 1 The results of HCT reverted to the outflow equation (1).

Watershed	Hydrograph error R^2 (%)	Flood peak Q_{max} error *PQ* (%)	Flood volume error *IVF*
Mingli	99.419	99.953	0.0573
Jiaohe	99.434	99.642	0.0482
Hengdaozi	99.298	99.751	0.0587

IVF: index of volumetric fit.

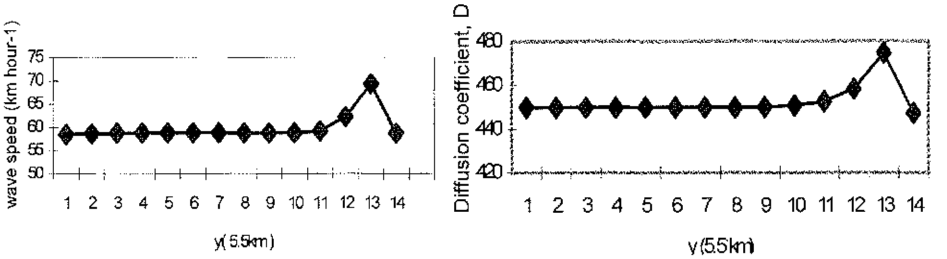

Fig. 3 (a) distributed wave speed and (b) distributed diffusion coefficient.

SUMMARY

Before a physically-based distributed catchment model can be used with confidence, it must be demonstrated that the performance is adequate and that the preparation and calibration of the parameter sets do not pose overwhelming practical difficulties. Owing to the lack of more comprehensive field data on real large and medium-sized catchments, there are few records of the actual performance of physically-based models. Unit-width discharge data from overground and underground are not measured in the field. Currently, it is nearly impossible to get the observed distributed information to calibrate the models, let alone to get the accurate distributed information itself. The hillslope is usually of a spatial scale appropriate to the small basin studies. The proposed inverse model can provide the information such as the unit-width flows, and all spatio-temporal parameters in each cell from hydrological records made at the mouth of the channel.

A one-dimensional hydrodynamic and numerical inverse model for the unit-width flows and all spatio-temporal parameters has been presented. By applying a counter-distributed and counter-boundary hybrid control technique (HCT) in simulating storm events (1958–1995) in Mingli, Jiaohe and Hengdaozi catchments in northeastern China, encouraging results have been obtained when tested against hydrological data restoration tests. The information of unit-width discharge can been applied to basins for calibration of spatio-temporal parameters. A more detailed discussion about the spatio-temporal parameters and the accuracy of simulation will be presented in a separate paper.

Acknowledgement The author is grateful to Shenglian Guo for total financial support and hydrologic data from Feng Man Reservoir.

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