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A distributed dynamic parameters inverse model for rainfall–runoff

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Abstract A distributed and dynamic parameters inverse technique is proposed, by which a physics-based rainfall–runoff model is based on hillslope hydrology, soil moisture and groundwater hydrodynamics, and open-channel hydraulics together with mathematical physical inverse theory. The model couples one-dimensional transient subsurface flow (soil evaporation, saturated and unsaturated soil water flow), and one-dimensional unsteady overland flow (vegetation interception, infiltration, evaporation and transpiration), with groundwater flow (infiltration and evaporation). The inverse model allows spatial distribution and time diversification in the hillslope, soil moisture and groundwater physical characteristics, including vegetation interception, infiltration, evaporation, soil hydraulic conductivity, overland flow wave speed, subsurface flow wave speed, groundwater wave speed, etc. Using the inverse formulas of the physical variables and parameters provides a dynamic, distributed, nonlinear, and stabilized model. The model uses a finite difference solution method with a mathematical physical inverse method. The model and inverse technique have been applied to three sub-basins within Feng Man Reservoir basin, China. The results indicated that the inverse technique works quite well in rainfall–runoff simulation for large and medium catchments using 33 years of flood season hydrological records from the mouth of the channel.

Key words rainfall–runoff model; inverse technique; finite difference method; Feng Man Reservoir catchment; pulse spectrum method; flood season

INTRODUCTION

The model comprises four subsystems coupled together: (a) a lateral inflow and channel-head inflow counter-distributed and converse boundary hybrid control model; (b) a normal rainfall–runoff model; (c) a distributed and dynamic parameters inverse model; and (d) a real-time flood forecast model.

This paper develops a distributed and dynamic normal rainfall–runoff coupled and inverse model. The model can simulate hillslope runoff processes and soil moisture flux processes in time and space. Based on the current state-of-the-art distributed rainfall–runoff models, a counter-time and converse-boundary hybrid control method and a distributed and dynamic parameters inverse algorithm are presented in this study. To solve the model equation, the technology was tested successfully on three different sub-basins (52 200, 108 500 and 250 000 ha, respectively) of Feng Man basin in China for 33 years of flood season hydrological records from the mouth of each sub-basin channel. For each of the three sub-basins, the temporal and spatial relationships between adjacent grid cells were taken into account utilizing the mathematical and physical coupling model and parameter inverse technique. The equations of the model were based on hillslope hydrology, soil moisture and groundwater hydrodynamics, and open-channel hydraulics theory. The structure of the model is correct and the technology is practical as shown in the examples. Some hydrology laws have been analysed by inverting the temporal and spatial parameters and variables. The predicted results can be improved when the hydrological laws are applied to flood forecasts.

NORMAL PHYSICS-BASED RAINFALL–RUNOFF MODEL

The distributed and dynamic hydrological model includes hillslope runoff, soil moisture flux and groundwater flow at a range of space and time scales. It represents the following processes: surface runoff, subsurface flow, recession flow, saturation overland flow, phreatic groundwater flow, and channel runoff, with their spatial distribution represented by partial differential equations. The input term (such as rainfall excess) changes in time and space. The model can also handle unsteady or complex rainfall excess and soil water outflow processes, such as infiltration water in the vadose zone in a perched aquifer.

Two rectangular planes plus a V-shaped channel form the geometric configuration used in the watershed model representing distributed hydrological processes in space and time. The kinematic equations for excess and saturation overland flow, as derived by the continuity and kinematic momentum equations, are:

$$\frac{\partial q}{\partial t} + v_1 \frac{\partial q}{\partial x} = v_1 [i(x,t) - i_r(x,t) - f_1(x,t) - E_1(x,t)] \quad (1)$$

and

$$q(s, t) = q(x, t) \times \cos(\theta)$$

where q is the overland runoff, consisting of surface runoff and saturated overland flow ($m^2\ km^{-1}$), v_1 is the wave speed of the overland runoff ($m\ h^{-1}$), i is the rainfall ($mm\ h^{-1}$), i_r is the evaporation of intercepted rainfall from the canopy ($mm\ h^{-1}$), f_1 is the actual infiltration ($mm\ h^{-1}$) of depression storage on the hillslope, and E_1 is the evapotranspiration ($mm\ h^{-1}$) from the land surface.

The physics of flow through porous media has been studied in great detail by groundwater hydrologists and soil physicists. The equation of subsurface flow q_w, developed from the continuity and kinematic momentum equations, becomes:

$$\frac{\partial q_w}{\partial t} + k_1 q_w + \omega_1 \left[\frac{\partial q_w}{\partial x} \right] = \omega_1 [f_2(x, z_{i-1}, t) - f_2(x, z_i, t) - E_2(x, z_i, t)] \quad (2)$$

$$s_1 \frac{\partial h_w}{\partial t} + \frac{\partial q_w}{\partial x} = f_2(x, z_{i-1}, t) - f_2(x, z_i, t) - E_2(x, z_i, t) \quad (3)$$

where s_1 is the transmissivity of the aquifer, q_w is the subsurface runoff, f_2 is the actual infiltration ($mm\ h^{-1}$) of the soil aquifer, ω_1 is the wave speed of the subsurface flow, $h = \varphi - z$, where φ is the total hydraulic head and z is the elevation head, and E_2 is the soil evaporation.

The regression flow equation can be described as follows:

$$\frac{\partial q_w}{\partial t}+k_2\frac{\partial q}{\partial t}+v_2[\frac{\partial q_w}{\partial x}]=v_2[f_1(x,t)-f_2(x,z_1,t)-E_2(x,z_1,t)] \tag{4}$$

and

$$q_w(s, t) = q_w(x, t) \times \cos(\theta)$$

where v_2 is the wave speed of the regression flow and θ is the slope gradient of the bedrock.

The groundwater flow is given by the groundwater continuity equation:

$$\frac{\partial q_g}{\partial t}+\omega\frac{\partial q_g}{\partial x}+k_g q_g=\omega[f_2(x,z_{m-1},t)-E_3(x,t)] \tag{5}$$

and the momentum equation:

$$q_g(s, t) = q_g(x, t) \times \cos(\theta)$$

where q_g is the groundwater flow, ω is the groundwater wave speed, f_2 is the actual infiltration at the aquifer when z is equal to l_z (where $l_z = z_{max} - z_0$, z_{max} is the elevation of the land surface, z_0 is the elevation of the bedrock), and E_3 is the evaporation from groundwater.

Channel flow can be described by the one-dimensional hydrodynamic equations:

$$\frac{\partial Q}{\partial y}+\frac{\partial A}{\partial t}=q_u \tag{6}$$

$$q_u(t) = q(s_1, t) + q_w(s_1, t) + q_g(s_1, t) \tag{7}$$

and the momentum equations for unsteady, non-uniform, spatially-varied flow in an open-channel:

$$\frac{v}{g}\frac{\partial v}{\partial y}+\frac{1}{g}\frac{\partial v}{\partial t}+\frac{\partial h_s}{\partial y}-(s_0-s_f)-(\frac{q_u v_{qy}-q_u v_q}{gA})=0 \tag{8}$$

The surface runoff equation is described by:

$$\frac{\partial Q}{\partial t}+c(y,t)\frac{\partial Q}{\partial y}-d(y,t)\frac{\partial^2 Q}{\partial y^2}=-d(y,t)\frac{\partial q_u}{\partial y}+c(y,t)q_u \tag{9}$$

Equations (1)–(9) with the preceding initial and boundary conditions can be solved using numerical techniques applied to field data. In using the hydrodynamic model to calculate the flow process of confluence, it is first necessary to calibrate the parameters and variables of the model (such as hydraulic conductivity, specific wave velocity of overland flow, channel wave speed and diffusion coefficient) and identify the state variables (such as those representing the infiltration, evaporation and transpiration processes) with the inverse model against the field recorded data. The derivation of land surface parameters and part state variables is inevitably very complicated due to the nonlinear, unstable and interacting processes involved. The inverse solutions come from the inverse technique and stability algorithm directly based on the physics-based rainfall–runoff model equations. There are distributed, nonlinear, unstable and dynamic calculation problems from the inverse calculation for the temporal and spatial parameters. The distributed problem is solved using the pulse spectrum technique (PST)

(Li, 1994, 1995), an unstable problem is treated using the regularization method, and a dynamic calculation problem is treated by gliding the m groups of data before time, t (Li, 1998). The approach was effective in determining the temporal spatial parameters and state variables using basin outlet record channel flow data.

The numerical experiment described in this paper uses data from three sub-basins of Feng Man reservoir basin. Using hourly time steps (Δt = 6 h), the flow region is modelled as a grid of points separated by a finite distance, where Δx = 2.5 km is the horizontal space step, and Δy = 5.5 km is the longitudinal step along the river. The size of the nodal area is determined mainly by the hydrological characteristics and optimization in the computational segment.

Distributed and dynamic parameter inverse

Based on hillslope hydrology, soil moisture and groundwater hydrodynamics, and open-channel hydraulics theory, the large-scale hydrological parameters and variables can be estimated using mathematical physical inverse problem equations and an inverse calculation algorithm. The problem suggests a physically-based rainfall–runoff inverse model with temporal and spatial parameters and variables, and provides the inverse formulations of the distributed dynamic parameters and variables, viz. surface runoff, saturated overland flow, regression flow, subsurface flow and groundwater flow.

Initial conditions

$$\begin{aligned} &q(x, 0) = 0 \\ &q_w(x, 0) = 0 \\ &q_g(x, 0) = q_u(0) \end{aligned} \tag{10}$$

Boundary conditions

$$\begin{aligned} &q(0, t) = 0.0 \\ &q_w(0, t) = 0.0 \\ &q_g(0, t) = C_1 \end{aligned} \tag{11}$$

where C_1 is the groundwater inflow from outside the border of the drainage basin.

Additional boundary conditions at the foot of the hill slope are:

$$q(s_n,t)+q_w(s_n,t)+q_g(s_n,t)=q_u(s_n,t) \tag{12}$$

where s_n is the local surface slope distance (m).

The first kind of the Fredholm integral equations of overland flow becomes:

$$q_s(l,t)-q(l,t) = \iint G_p(x,t,\alpha,\beta)\left\{-\delta k_1\left[\frac{\partial q}{\partial x}-r\right]+\delta f\left[k_1/3600\right]\right\}\mathrm{d}x\mathrm{d}t \tag{13}$$

where G_p is the Green function of overland flow:

$$\frac{\partial G_p}{\partial t}+k_1\frac{\partial G_p}{\partial x}=\delta(x,t;y,\xi) \tag{14}$$

and

$$\delta(x,t;y,\xi)=\begin{cases}1, x=y, t=\xi\\ 0, x\neq y \text{ or } t\neq\xi\end{cases} \tag{15}$$

For resolving as a nonlinear problem, the iterative formula is:

$$\begin{cases}K_1^{n+1}(x,t)=K_1^n(x,t)+\delta K_1^n(x,t)\\ f^{n+1}(x,t)=f^n(x,t)+\delta f\ (x,t)\\ q^{n+1}(x,t)=q^n(x,t)+\delta q^n(x,t)\end{cases} \tag{16}$$

where K_1^0, f^0, and q^0 are the initial values. The temporal spatial parameters and variables can now be reformulated by taking the term $K_{1ij}^{n+1}, f_{ij}^{n+1}$ into equation (16) to obtain the numerical solution of the overland flow and saturated overland flow processes.

For the temporal and spatial parameters and variables of subsurface flow and groundwater flow, the Green function of subsurface flow is needed using the PST method. The Green function formula is:

$$\frac{\partial G_w}{\partial t}+\frac{k_{21}}{k_1}\frac{\partial q}{\partial t}+\frac{k_{21}}{s_1}\frac{\partial G_w}{\partial x}=\delta(x,t;y,\xi) \tag{17}$$

The first kind of the Fredholm integral equations of overland flow can be written as:

$$q_w^s(l,t)-q_w(l,t)=\iint G_w(x,t,\alpha,\beta)\left\{-\delta k_{21}\left[\frac{1}{k_1}\frac{\partial q}{\partial t}+\frac{1}{s_1}\frac{\partial q_w}{\partial t}-\frac{r_w}{s_1}\right]\right\}\mathrm{d}x\mathrm{d}t \tag{18}$$

For resolving as a nonlinear problem, the iterative formula is:

$$\begin{cases}K_{21}^{n+1}(x,t)=K_{21}^n(x,t)+\delta K_{21}^n(x,t)\\ q_w^{n+1}(x,t)=q_w^n(x,t)+\delta q_w^n(x,t)\end{cases} \tag{19}$$

Using these relationships, the other groundwater parameters can be expressed as:

$$\begin{cases}K^{n+1}(x,t)=K_2^{n+1}(x,t)\cdot\Delta y/\Delta x\\ K_4^{n+1}(x,t)=K^{n+1}(x,t)/S_2\\ K_3^{n+1}(x,t)=K_2^{n+1}(x,t)/K_1^{n+1}(x,t)\end{cases} \tag{20}$$

The temporal spatial parameters and variables can now be reformulated by moving $K_{21ij}^{n+1}, K_{3ij}^{n+1}, f_{ij}^{n+1}, K_{4ij}^{n+1}$ into equations (2), (4) and (5) to obtain the numerical solution of subsurface processes, regression flow processes, and phreatic groundwater flow processes.

RESULTS AND DISCUSSION

Thirty-three years (between 1958 and 1995) of flood season hydrological records from the mouth of the channel were chosen for simulation. The modelled areas are the three sub-basins Mingli, Jiaohe, Hengdaozi, respectively, of the Feng Man reservoir basin in northeast China. The three rivers flow into the upper reaches of the Songhuajiang River at different locations. The length of Mingli River is 50 km, the Jiaohe River is 63 km, and the Hengdaozi is 40 km. The average annual precipitation for the three areas of the basin is about 750 mm. The total area of the Feng Mang reservoir basin is about 4 250 000 ha, of which the Mingli sub-basin is about 108 500 ha, the Jiaohe sub-basin is 250 000 ha, and the Hengdaozi sub-basin is 52 200 ha. The flood data were sampled at 6-h intervals.

For the 33 years of flood season hydrological records for which discharge at the mouth of the channel was simulated, the average value of R^2 (Nash-Sutcliffe coefficient) was 98.2, 88 and 95.6% in the Mingli, Jiaohe, and Hengdaozi sub-basins, respectively. The average index of volumetric fit (*IVF*) was 0.053, 0.036 and 0.082, respectively. The term *PQ* expresses the average eligible percentage of relative peak errors. Relative peak errors are between 0 and 15% so that the simulative peak is eligible. The *PQ* values for the three sub-basins studied were 96.4, 92.2 and 97.4%, respectively.

Representing a variety of rainfall events and catchment responses, some of the test runs are illustrated in Table 1 and Figs 1–3. All simulations were made for the flood period 1 June–30 September each year.

In all cases the observed and simulated hydrographs are reasonably well matched (see Table 1). Since calibration of the model parameters was carried out for individual events, the results both validate the model and show that an effective calibration can be based on raw data. The results also illustrate the ability of the inverse model to deal satisfactorily with the varied responses of different parts of the catchments under different rainfall conditions using the catchment parameters. In particular, for storms with double peaks (all large events) the model is able to model the interstorm period sufficiently well for the correct initial conditions for a satisfactory simulation of the second peak to be achieved.

Table 1 Modelling results for three sub-basins (1958–1995).

Sub-basin	Event year	R^2(%)	*IVF*	*PQ*(%)
Mingli	Wet year	98.8	0.0609	94.95
	Median year	98.0	0.0687	99.17
	Dry year	97.9	0.0293	95.12
	Average	98.2	0.0530	96.41
Jiaohe	Wet year	93.3	0.0198	94.12
	Median year	84.0	0.0727	96.00
	Dry year	86.6	0.1537	86.61
	Average	88.0	0.0821	92.24
Hengdaozi	Wet year	98.4	0.0206	94.19
	Median year	96.2	0.0374	99.21
	Dry year	92.2	0.0489	98.84
	Average	95.6	0.0356	97.41

The relationships for wave speed and rainfall were found by analysing the inverse temporal spatial parameters and the observed data of rainfall intensity. For example, some relationships of Mingli were given with the inverse parameter values and the observed rainfall data. The wave speeds are affected by the size of the time and space steps used in a model run (see equations (21), (22) and (23) below). One of the relationships developed between overland flow wave speed, υ_1, and rainfall intensity, I, is:

$$v_1 = 0.0138I + 0.0682 \quad (21)$$

where the correlation coefficient, $R = 0.81$. The subsurface flow wave speed, v_2, is related to the rainfall mass curve, ΣP:

$$v_2 = 0.0257\Sigma P - 0.3971 \quad (22)$$

where $R = 0.75$. The third relationship is between groundwater outflow speed, ω, and rainfall mass curve, ΣP:

$$\omega = 0.028\Sigma P - 0.0428 \quad (23)$$

where $R = 0.75$.

All kinds of wave speed distribution are a function of space. The function curves are shown in Fig. 1(a), (b) and (c), respectively.

Figure 2 shows the groundwater flow process from a rainfall–runoff event. Figure 3(a)–(c) shows the overland flow, subsurface flow and groundwater flow

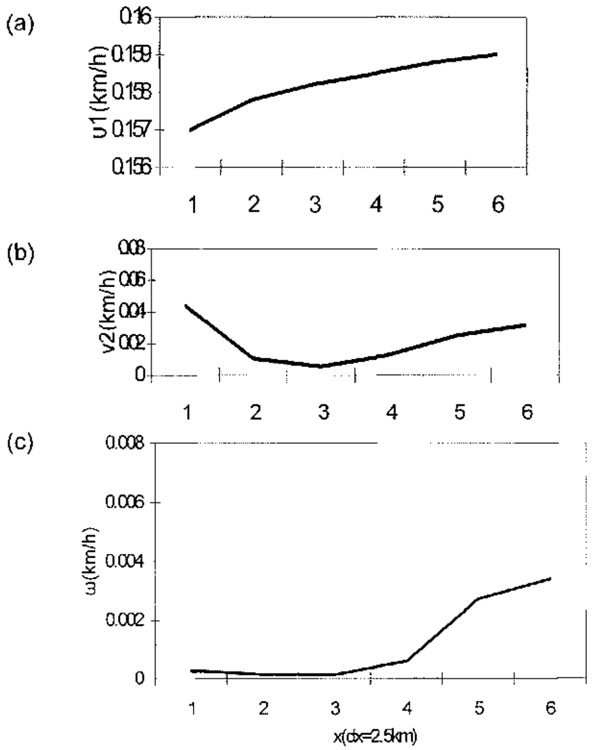

Fig. 1 Spatial distribution of (a) overland wave speed; (b) subsurface outflow speed, and (c) groundwater outflow speed.

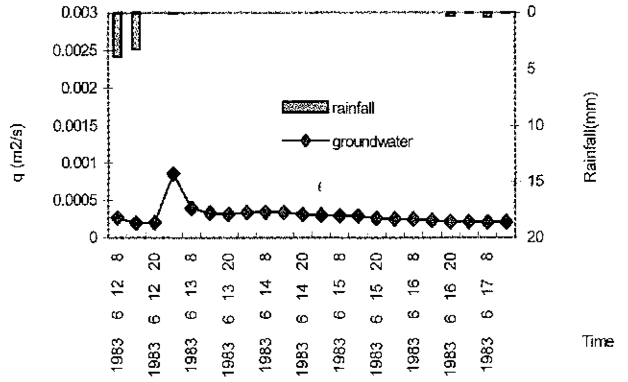

Fig. 2 Groundwater flow processes.

processes for the 1 June–30 September flood season in 1983 and 1995 in the Mingli sub-basin. Figure 3 (d) shows some infiltration process from the same period.

CONCLUSIONS

All the watershed variables and parameters can vary significantly in space and time due to the variability in hydrological properties. Current hydrological parameterizations assume stationary and average system in their calibrations. A distributed numerical solution was given in a hydrological distributed and dynamic model. It can represent multiple runoff simulation processes in the each time node and at the each spatial node. By applying it to a set of rainfall–runoff events on three natural watersheds, it has been demonstrated that this approach is computationally far more efficient than a integrated model. Outflow to the channel is the result of lateral inflows including overland flow, baseflow, and interflow. The channel flow record during at period without rainfall can be used to estimate the parameters and variables of the groundwater flow. Overland flow can be identified by discharge information from the beginning of the flood to its peak interval. The relationships of distributing and time-varying parameters and variables were identified based on median unit flow values of the input channel. The predicted results can be improved when the relationships was applied to flood forecasts, offering reasonably accurate predictions and model run times.

Models of the hydrology of the land phase, however, are generally only available at a smaller basin scale. But in this research, a model has been developed at a continental-scale into components suitable for a medium or large catchment. It is possible to using the temporal and spatial parameters and variables to improve the model simulation. This paper has demonstrated the practical implementation of a physically-based, distributed and time-varying catchment model and its ability to simulate a variety of catchment responses at large natural basin.

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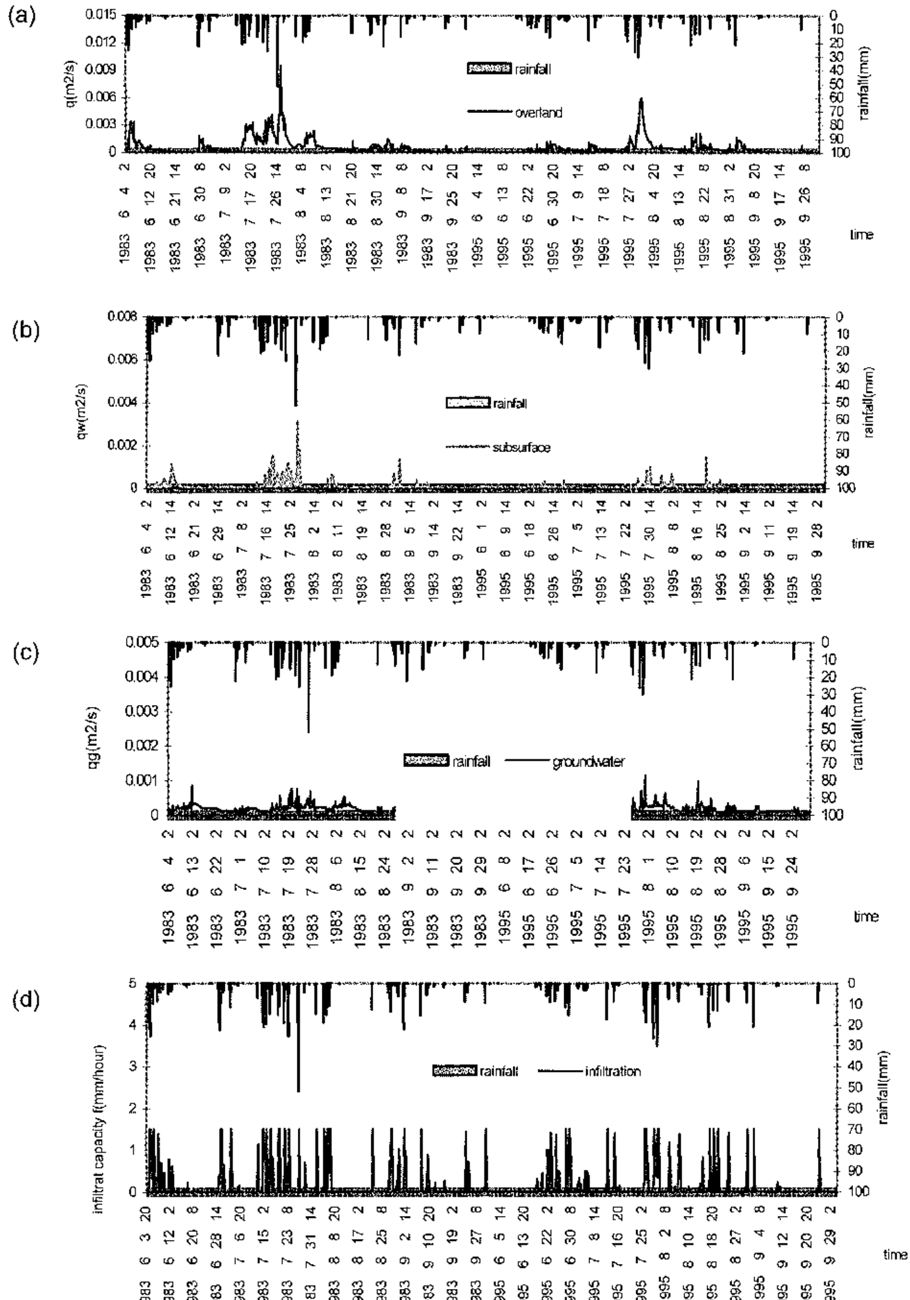

Fig. 3 (a) Overland flow, (b) subsurface flow, (c) groundwater outflow and (d) infiltration processes for 1983 and 1995.

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