

The method of impulse response moments: a new method integrating time series-, groundwater- and eco-hydrological modelling

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Abstract The impulse response (IR) function describes the way in which the water table responds to an impulse of precipitation, and plays an important role in time series analysis. The moments of the IR function can, however, be directly calculated from both time series models and groundwater models, and can also be used in eco-hydrological models as moments characterize the dynamic behaviour of the water table. In this paper the outline of the method of impulse response moments (IRM) is presented. This method integrates the above mentioned fields of hydrological modelling as subsystems from which moments can be derived and between which moments can be mutually exchanged, thereby offering an opportunity to combine their specific advantages. Future research will focus on the further development of the IRM method and on quantifying the consequences of the assumptions made (i.e. linearity of the hydrological system and a predefined form of the IR function).

Key words eco-hydrology; groundwater level fluctuations; groundwater modelling; method of impulse response moments; time series modelling; water table dynamics

INTRODUCTION

In a large part of The Netherlands the water table is at shallow depths. In about 50% of Dutch soils, the mean highest water table depth (MHW) is less than 40 cm below the soil surface (Van der Sluijs, 1990). As the groundwater regime can have a large influence on site conditions and soil formation, it can also directly influence the functioning of a site with respect to its economic and/or ecological use. Therefore knowledge about the temporal and spatial variation of the groundwater level is important.

Transfer Function Noise (TFN) time series models are often used to model time series of water table depth by relating them to precipitation excess series (Van Geer *et al.*, 1988; Rolf, 1989; Gehrels, 1999). Long-term measurements from the sites in question must be available to be able to use the entirely empirical TFN models, so no assessment can be made of the consequences of recent or future hydrological interventions. Distributed transient groundwater models are physically based, and can be used to model the spatio-temporal variation of the groundwater level for land-use or water management planning. Forecasts from time series models are, on the other hand, mostly more accurate than costly, complex conceptual models (Hipel & McLeod, 1994).

In most eco-hydrological models, the relationship between the groundwater regime and the composition of the natural vegetation is modelled using simple statistics like the mean highest, lowest and spring water table depths, MHW, MLW and MSW respectively, together referred to as MXW values (Grootjans *et al.*, 1996; Olde Venterink & Wassen, 1997). MXW values are derived from measured, mostly short, time series of water table depth. They are sensitive to variations in meteorological circumstances, and should therefore be calculated on the basis of very long time series (30 years), whether or not extended with the aid of time series models (Knotters & Van Walsum, 1997).

THE IMPULSE RESPONSE FUNCTION

The impulse response (IR) function describes the way in which the water table responds to an impulse of precipitation. In that respect it is similar to the instantaneous unit hydrograph used in surface water hydrology. A typical IR function looks like a very askew probability distribution function (Fig. 1). The form and area of the impulse response function depend strongly on the hydrological circumstances *in situ*. Where, for instance, the flow resistance to the nearest ditch or means of drainage is low, the water table will drop quickly after a shower and consequently the area of the IR function will be small.

From a mathematical point of view, a time series of water table depth is a transformation of a time series of precipitation surplus. Under the assumption of linearity, the transformation of a precipitation surplus series into a series of water table depth is completely governed by the IR function. Written as a convolution integral:

$$h(t) = d + \int_0^t p(t - \tau)\theta(\tau)d\tau \quad (1)$$

where h is the course of the water table depth, d denotes the local drainage level, i.e. the level to which the water table declines without precipitation, p is the time series of precipitation surplus and θ is the IR function. Moments are widely used in the field of statistics to characterize probability density functions (PDF). Likewise, moments can

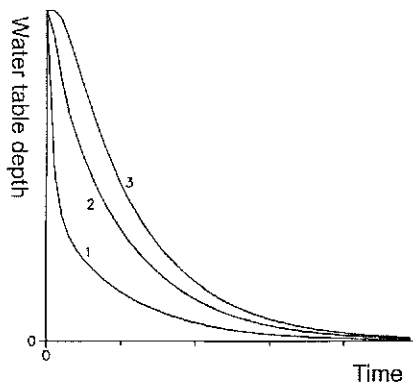


Fig. 1 Response of the water table to a pulse of rain: (1) near a ditch, (2) at some distance from a ditch, and (3) far from a ditch.

also be used to characterize the IR function in convolution processes (Maas, 1994). Moments of higher order add extra information about the form of a function to the lower order moments, the function being defined exactly with an infinite number of moments. The general equation of a moment M of order n is:

$$M_n = \int_0^{\infty} t^n \theta(t) dt \quad (2)$$

THE METHOD OF IMPULSE RESPONSE MOMENTS

The method of impulse response moments (IRM) combines three fields of hydrological modelling. Firstly, the impulse response function plays an important role in time series analysis, as it follows directly from the coefficients of a time series model (Box & Jenkins, 1970). Secondly, moments can be calculated directly from the computer code of a numerical groundwater model such as MODFLOW, because the differential equation for moments is closely related to the differential equation for groundwater flow (Lankester & Maas, 1996). Thirdly, moments play a role in eco-hydrological modelling because the relationship between the hydrological circumstances and the composition of the natural vegetation can be characterized by moments.

These three modelling fields form subsystems of the IRM method presented here. From these subsystems, moments can be derived and mutually exchanged (Fig. 2). The way in which moments can be used in and calculated from every subsystem is described below.

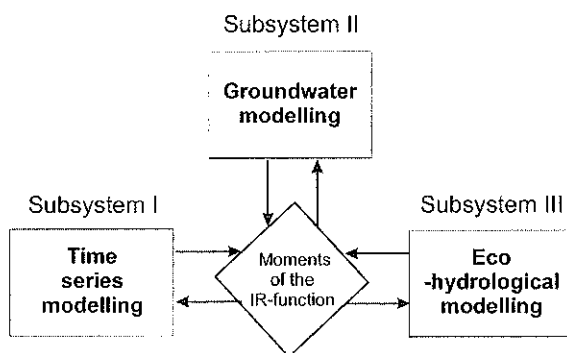


Fig. 2 Overview of the IRM method, showing the three subsystems from which moments can be derived and between which moments can be mutually exchanged.

Subsystem I: Time series modelling

Figure 3 gives an overview of the role that moments play in time series modelling within the IRM method. As in TFN models, the water table depth is modelled here by using a precipitation surplus series as input. A white noise series is used to explain the differences between measured and simulated water table depth. From any given time series model an IR function and its moments can be derived from the numerical values

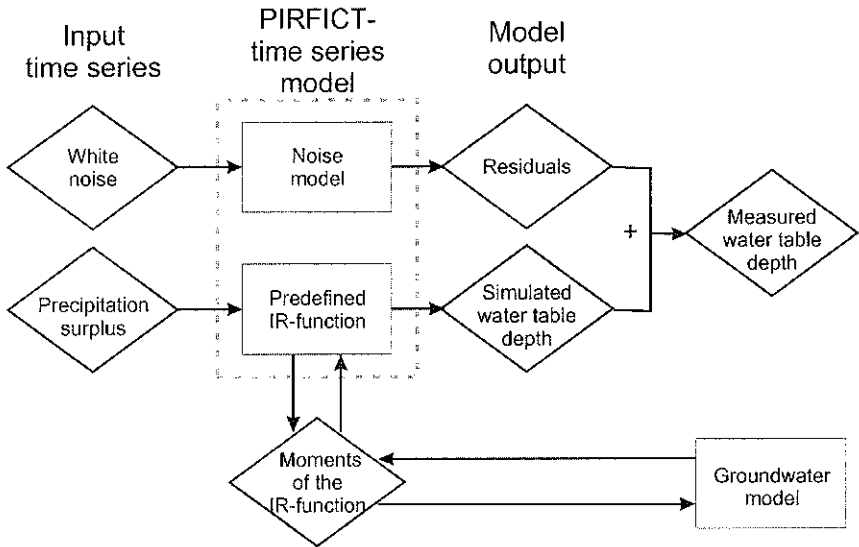


Fig. 3 Overview of the time series modelling subsystem.

of its parameters. However, to be able to construct a time series model from moments obtained from one of the other subsystems, the IR function should have a predefined form that can be formulated in terms of moments. For this reason a new type of time series model, the PIRFICT model, has been constructed which is based on continuous IR functions. PIRFICT stands for Predefined IR Function In Continuous Time. When combining equation (1) as a transfer model with a noise model in the form of a convolution integral, the general equation of a PIRFICT time series model is obtained:

$$\tilde{h}(t) = \int_0^t \tilde{p}(t - \tau) \theta(\tau) d\tau + \int_0^t a(t - \tau) \phi(\tau) d\tau \quad (3)$$

with $\tilde{h}(t)$ = deviations of the water table depth series from the mean, $\tilde{p}(t)$ = deviation of the precipitation surplus series from the mean, $a(t)$ = the white noise input series and $\theta(t)$, $\phi(t)$ = a predefined impulse response function, e.g. of the form:

$$\theta(t) = A * \text{Peason type III PDF} = A \frac{a^n t^{n-1} \exp(-at)}{\Gamma(n)} \quad (4)$$

with A , a and n being constants. Using equation (2), for a PIRFICT model with θ conform equation (4), A , a and n are related to the n th, and the first three moments in the following manner (Maas, 1994):

$$M_n = \int_0^\infty t^n A \frac{a^n t^{n-1} \exp(-at)}{\Gamma(n)} dt, \quad M_0 = A, \quad M_1 = A \frac{n}{a}, \quad M_2 = A \frac{n}{a^2} (1+n) \quad (5)$$

Because the PIRFICT model is defined as a continuous function, any time discretization can be used to solve the equations. Furthermore, the model separates the time steps of the output variable t from the time steps of the input variable τ . Thus $p(t - \tau)$ can be given in days, while the times steps at which $h(t)$ is given

any other discretization, even an irregular one, thereby accounting for the often non-equidistant time steps of most series of water table depth. With normal TFN models, a Kalman filter algorithm is necessary to handle sparsely or irregularly observed time series (Bierkens *et al.*, 1999).

Subsystem II: Groundwater modelling

Using a distributed stationary groundwater model such as MODFLOW, the moments of the IR function can be calculated in a way that closely resembles the calculation of water table depths. This can be derived from the general equation of groundwater flow. For the simple case of a homogeneous aquifer, the equation for the groundwater head is:

$$\nabla^2 h(t) = \frac{\varepsilon}{kD} \frac{\partial h(t)}{\partial t} - \frac{p(t)}{kD} \quad (6)$$

with h being the groundwater head, ε the phreatic storage coefficient, kD the transmissivity and p the precipitation surplus. If $p(t)$ is an impulse of unit volume at $t = 0$, the course of h is by definition the IR function θ , and $p(t)$ can be substituted by $P_0 \delta(t)$. P_0 denotes the volume of the precipitation that has fallen instantaneously on $t = 0$, which for an IR function is equal to 1, and $\delta(t)$ denotes the Dirac delta function. In order to obtain the n -th moment, both sides of equation (6) are multiplied with t^n and integrated with respect to time from $-\infty$ to $+\infty$:

$$\nabla^2 \int_{-\infty}^{\infty} t^n \theta(t) dt = \int_{-\infty}^{\infty} t^n \frac{\varepsilon}{kD} \frac{\partial \theta(t)}{\partial t} dt - \frac{P_0}{kD} \int_{-\infty}^{\infty} t^n \delta(t) dt \quad (7)$$

This gives us, with the aid of equation (2) (Lankester & Maas, 1996):

$$\nabla^2 M_0 = -\frac{P_0}{kD} \quad (n = 0), \quad \nabla^2 M_n = -\frac{\varepsilon}{kD} M_{n-1}, \quad (n \geq 1) \quad (8)$$

Note that the factor time is not present in equation (8). For $n = 0$, equation (8) is mathematically identical to the equation of stationary groundwater flow and can be solved with any ordinary distributed groundwater model. For higher orders of n , M_n can be solved by using the moment of order $n - 1$ as input.

Van de Vliet & Boekelman (1998) used the distributed groundwater model TRIWACO to obtain M_0 and M_1 values from a dune area in The Netherlands. Figure 4 shows an east to west cross-section of the area, showing the surface level and the course of M_0 , as calculated by TRIWACO. The M_0 values obtained directly from the groundwater model proved to be comparable (the differences had a σ of less than 5%) to the M_0 values obtained from time series of water table depth that were generated by the same groundwater model. The results of M_1 were not comparable, due to the way in which the unsaturated zone influences the calculation of M_1 in time series models.

Subsystem III: Eco-hydrological modelling

As plant species have adapted to survive different abiotic circumstances, parameters correlated with abiotic circumstances that influence the fitness of plant species, often

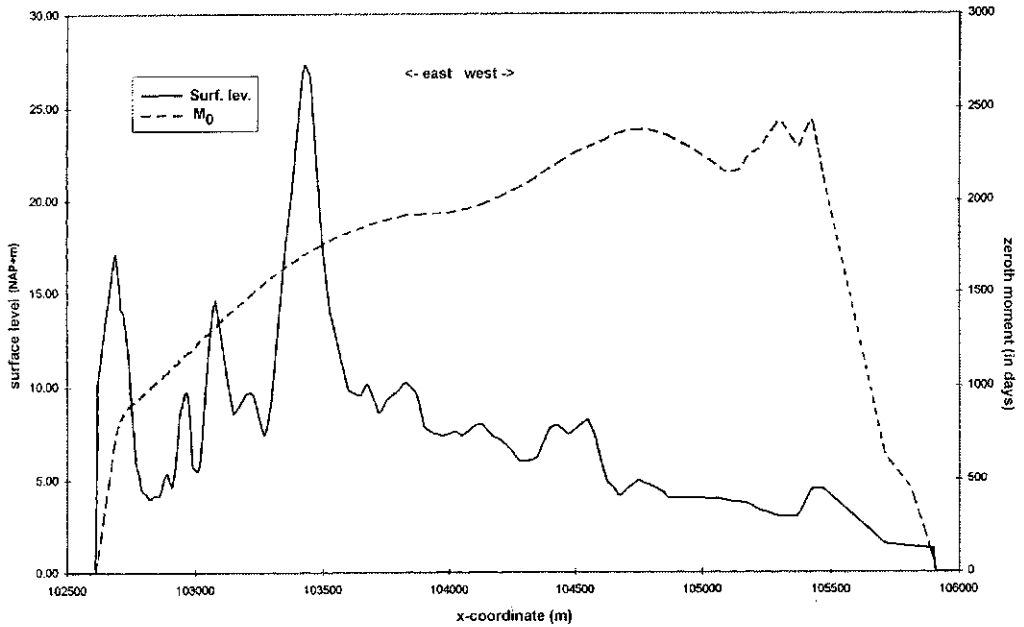


Fig. 4 East to west cross-section of the dune area near Wijk aan Zee (The Netherlands), showing the surface level and the course of M_0 (Van de Vliet & Boekelman, 1998).

referred to as “site factors”, have a predictive power with respect to the vegetation composition. Vice versa, the vegetation can give an indication on the prevailing abiotic conditions (Ellenberg, 1991).

Many eco-hydrological studies have shown that the groundwater regime can have a strong impact on site conditions and a strong correlation with the vegetation composition. From the combination of equations (1), (2) and (8), it follows that, under the assumption of linearity, the groundwater regime of an area $h(x,y,t)$ is fully determined by the spatial variation of the impulse response moments $M_n(x,y)$ of that area and the precipitation surplus series $p(t)$, which is more or less independent of x and y . This leads to the following correlation equation which can be used for eco-hydrological modelling:

$$h(t,x,y) \sim M_n(x,y) \sim F(k,x,y,t) \sim S(m,x,y,t) \quad (9)$$

with F denoting the state of site factor k and S denoting the abundance of plant species m , both as functions of x , y and t .

DISCUSSION AND CONCLUSIONS

As the techniques of time series-, groundwater- and eco-hydrological modelling separately are of course not new, the most important feature of the IRM method is the integration of these techniques, which offers an opportunity to combine their specific advantages. Thus, the IRM method could give a physical basis and a spatial

component to time series models of water table depth, thereby making it possible to extend their predictions beyond a specific site and the existing hydrological situation. Furthermore, it could bring some of the accuracy and ease of construction of time series models to groundwater modelling, by facilitating the calibration of transient groundwater models with measured time series and/or spatially modelling the dynamic characteristics of an area rather than actual time series. Last but not least, moments could capture the dynamic behaviour of the water table in a few, weather independent parameters, which could have a stronger relation with the composition of the natural vegetation than MXW values.

However, it remains to be seen what the consequences are of the assumptions made (i.e. linearity of the hydrological system and a predefined form of the IR function). The use of a predefined form of the IR-function could lower the performance of the PIRFICT model as compared to that of other TFN models. On the other hand, the performance is probably increased by the fact that the PIRFICT model can adequately handle non-equidistant and daily time series. In the research by Van de Vliet & Boekelman (1998) the results of M_1 obtained from a time series model and a groundwater model were not comparable. The current research focuses on improving the comparability of M_1 and higher order moments, and on comparing moments obtained independently from piezometers and a calibrated groundwater model.

A central hypothesis in the research on the IRM method is that moments have a stronger correlation with the composition of the natural vegetation than MXW values. Unlike MXW-values, moments are weather independent parameters, as they follow from the correlation between meteorological circumstances and water table depth, and under the assumption that the system is linear, the IR function and its moments are constant in time. The species composition of well-developed groundwater dependent vegetation is also weather independent, as it can be very stable for long periods. Furthermore, MXW values only describe the mean behaviour of the water table over a certain period, while the first two or three moments can describe groundwater level fluctuations in detail. Duration lines (Nieman, 1973) also characterize the course of the water table in more detail and are for that reason used in many eco-hydrological studies. Like MXW values, however, they are also influenced by meteorological fluctuations. Future research will have to show whether these more or less theoretical benefits of using IR moments indeed give rise to a stronger correlation with the vegetation composition.

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