Calibration of groundwater recharge and hydraulic conductivity for the aquifer system beneath the city of Milan (Italy)

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Abstract A quasi three-dimensional model has been developed to simulate the behaviour of the aquifer system, which is the source of drinking water for the city of Milan (Italy), and extends over an area of about 400 km^2. Non continuous semipermeable layers locally separate permeable levels in a multi-layered system, consisting of one phreatic and three confined aquifers. The numerical model is a conservative finite difference scheme, with a grid spacing of 500 m. Model calibration requires determination of the hydraulic conductivities of the aquifers and the aquitards, and of the parameters describing the source/sink terms. Water extraction rates from public and private wells are known, whereas estimates of recharge due to rain infiltration and losses from buried pipes, rivers and artificial channels are uncertain. Here we improve the identification of the hydraulic conductivities and perform a sensitivity analysis on some coefficients used to quantify the aquifer recharge.

Key words aquifer recharge; hydraulic conductivity; Milan (Italy); model calibration; multi-layered aquifer; sensitivity analysis

INTRODUCTION

The city of Milan takes water for domestic needs from the aquifer system located beneath the urban area. More than a thousand wells have been drilled in an area of about 400 km^2 (Fig. 1). The wells are owned by the municipal Water Works and are clustered around 34 pumping stations, each of which is serviced by a number of wells varying from four to 25.

Water extraction has varied during the twentieth century (Motta, 1989). The municipal Water Works extracted 7.6×10^6 m^3 of water in 1900. Water consumption increased linearly until 1940 and more rapidly after the Second World War. By that time, several industrial activities had developed, mainly in the northern suburbs of the city, and the population had increased. In 1974, the municipal Water Works extracted about 350×10^6 m^3. This caused a deep and wide depression cone around the city centre. In the last decades of the twentieth century, several big factories were relocated away from the city and the annual water consumption decreased to less than 300×10^6 m^3. This caused a rapid rise of the water table and, as a consequence, episodes of flooding of buried structures built or designed in the 1970s (e.g. garages, the underground railway).

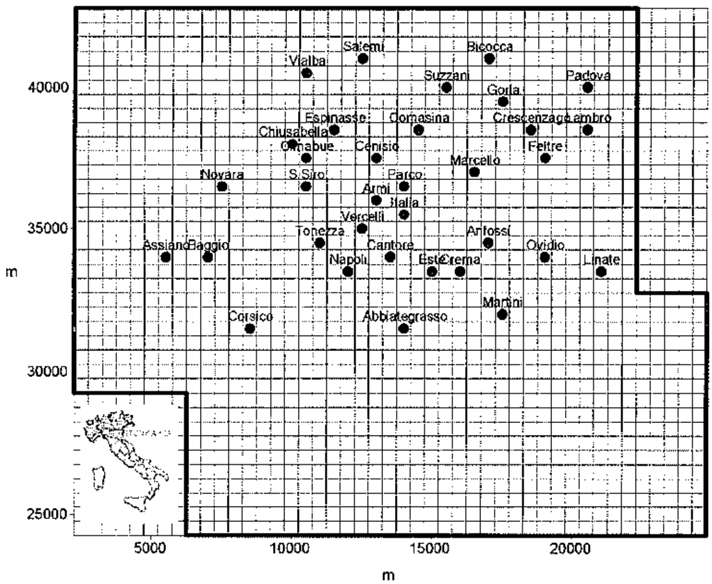

Fig. 1 Discretization grid and location of the Water Works well fields.

Giudici *et al.* (2000) have developed a quasi three-dimensional model of groundwater flow under stationary conditions to understand the physical behaviour of the aquifer system. This model refers to the "traditional aquifer" whose base is at a depth of about 100 m below the ground surface and which is the main source of drinking water for the city. The traditional aquifer can be modelled as a multi-layered aquifer, consisting of one phreatic and three confined leaky aquifers, which are separated from each other by discontinuous aquitards. The top of the first aquitard is located at an average depth of 30 m below the ground level. The average thickness of the confined aquifers varies between 14 and 20 m, whereas the thickness of the aquitards varies between 0 (at aquitard windows) and 15 m.

The groundwater flow model is based on a conservative finite differences scheme, which discretizes the integral mass balance equation over square cells, whose side length has been fixed at 500 m in accordance with the data distribution and with the goals of the model. In particular, in each cell of the model there is at most one Water Works pumping station. The main interest is the effect of the whole well field on the flow at large scale, rather than the local effect of a single well. The balance equations are nonlinear, since the transmissivity of the phreatic aquifer depends on the phreatic head. Moreover the balance equations to be solved are modified when the water table is lower than the bottom of the phreatic aquifer, which was the case in late 1970s.

The model was originally calibrated with a trial-and-error procedure applied to the data for the years 1950, 1974 and 1982. Flow conditions for these years were very different from each other; 1950 corresponds to the beginning of intensive exploitation of the aquifer, 1974 corresponds to the period of maximum exploitation and 1982 corresponds to the first rise of the water table.

Once calibrated, the model was used as a predictive tool to forecast the behaviour of the aquifer system for different years to those used in the calibration phase; the comparison between the model forecasts and available observations was good (Giudici *et al.*, 2000). However, despite the good quality of the results, some model parameters were still uncertain. In particular, the rates of recharge by rain infiltration, and due to losses from channels, rivers and buried pipes, are difficult to estimate. This paper focuses on an improvement of the calibration of hydraulic conductivities and a sensitivity analysis on the recharge terms.

PARAMETERIZATION OF HYDRAULIC CONDUCTIVITY AND SOURCE TERMS

Hydraulic conductivity of the aquifers and the aquitards

The four aquifers and the aquitards are assumed to be homogeneous. The same value of conductivity is assigned to all the aquitards; the hydraulic conductivity of the aquifers is assumed to decrease with depth, since lithological well logs show that compaction, cementation and presence of fine grained sediments increase with depth.

Let K_{ph}, K_{c1}, K_{c2}, K_{c3} and K_s denote the hydraulic conductivity of the four aquifers and of the aquitards, respectively. We parameterize K_{ph}, K_{c1}, K_{c2}, and K_{c3} with two parameters, K_{eq} and r, which correspond to the equivalent conductivity of the whole aquifer system and to the ratio between the conductivity of an aquifer and the conductivity of the underlying aquifer:

$$K_{eq} = \left(K_{ph} + K_{c1} + K_{c2} + K_{c3}\right)/4 \tag{1}$$

$$K_{ph} : K_{c1} : K_{c2} : K_{c3} = r^3 : r^2 : r : 1 \tag{2}$$

Equations (1) and (2) are the simplest choices compatible with the available data and information.

If the values of K_{eq} and r are assigned, K_{ph}, K_{c1}, K_{c2} and K_{c3} can be computed with equations (1) and (2). The calibration performed by Giudici *et al.* (2000) led to $K_{eq} = 2 \times 10^{-3}$ m s^{-1} and $r = 5$, which correspond to $K_{ph} = 6.4 \times 10^{-3}$ m s^{-1}, $K_{c1} = 1.3 \times 10^{-3}$ m s^{-1}, $K_{c2} = 2.6 \times 10^{-4}$ m s^{-1} and $K_{c3} = 5.1 \times 10^{-5}$ m s^{-1} The value of the hydraulic conductivity for the aquitards, $K_s = 1 \times 10^{-7}$ m s^{-1}, was assumed to be about 500 times smaller than K_{c3}.

Water extraction

The volume of the annual water extraction of each pumping station is available from the Municipal Water Works. However, the screens of each well are located at several different depths. Since the location of the screens, their characteristics and the depth of the pump are uncertain, particularly for old wells, the best estimate is to assume that the flow rate from each aquifer is proportional to the corresponding aquifer transmissivity. Unfortunately there is no definite answer to the question of whether these wells are screened even in the highly permeable phreatic aquifer. We consider the effect of alternative hypotheses on the phreatic head in the section below on the "Identification of hydraulic conductivity".

Annual water extraction from private wells is known and is subdivided among the administrative zones of the city; within each zone the flow rate is distributed homogeneously. The distribution of these terms among the four aquifers is done with the same approximation as for the public wells. Most of the private wells are probably shallow; however, since hydraulic conductivity decreases with depth, the assumption that water extraction by private wells from each aquifer is proportional to the corresponding aquifer transmissivity is acceptable.

Relations between rivers and the phreatic aquifer for natural flow conditions

Initially the model was used to improve the understanding of the relationship between the aquifer system and the main rivers and natural channels crossing the study area, by comparing the results obtained for the aquifer under undisturbed flow conditions and the historical information. In particular historical data from the nineteenth century show that the water table was a few metres below the ground surface throughout the whole area of the city. This is confirmed, for instance, by the existence of some pools even inside the city, where there were artificial or natural depressions.

If we run the model without sources terms due to water extraction from wells, which is a good approximation of the nineteenth century conditions, the water table is higher than the ground level in some places, which are in good correspondence with the main streams (e.g. Olona and Lambro Rivers). In this case, the rivers drain the aquifer and carry water downstream from the areas that appear "flooded". Therefore the model has been modified by adding an extraction source term, which simulates the river drainage and maintains the water table lower than the ground surface, in agreement with the available information. The drainage flow rates are consistent with the measured flow rates of the main streams, i.e. they are less then 30% of the average flow rates of the corresponding rivers.

At the present stage of modelling, we neglect losses from rivers and channels, because the available data are so uncertain that do not permit even a gross estimate; this is the subject of an ongoing research.

Parameterization of the phreatic aquifer recharge

The recharge of the phreatic aquifer is due to several terms: infiltration of rain water, losses from buried pipes (belonging to the distribution network of the Water Works and to the sewage network), losses from rivers, natural and artificial channels.

The precise location of the buried pipes is not available (for instance, the location of sewage pipes cannot be divulged for public security reasons); however the distribution of buried pipes is quite homogeneous over the whole modelled area. Furthermore, most of the area is urbanized and covered by roads, buildings and concrete; uncovered areas correspond to small parks in the city and some other areas in the suburbs. These remarks, and the dimension of the discretization cells, justified the choice of a homogeneous distribution of the recharge terms over the whole modelled area.

With the following equations, (3), (4) and (5), we introduce the parameterization of the recharge produced by infiltration of rain water, R_{rain}, losses from the Water

Works distribution network, R_{ww}, and losses from the sewage network, R_{sewage}. Let R denote the annual precipitation, which is close to 1000 mm and has shown small variations throughout the twentieth century; C_e denotes the fraction of rain that evaporates or is used by plants. Rain is largely collected by the sewage network and C_c denotes the fraction of collected rain water. The remaining part of the rain water infiltrates into the soil and C_i denotes the fraction of this water that can reach the water table. Therefore:

$$R_{rain} = R(1-C_e)(1-C_c)C_i \tag{3}$$

The water extracted by the Water Works wells flows in the distribution network and is partly lost. Let P_a denote the annual volume of water extracted by the municipal Water Works, divided by the surface of the modelled area, and C_{al} the fraction of water lost from the pipes of the distribution network. The recharge to the aquifer due to Water Works losses is given by:

$$R_{ww} = P_a\, C_{al}\, C_i \tag{4}$$

Pipes transferring water from private pumping wells to the final user are short, since these wells are normally drilled in the locality where the water is needed. Therefore these pipes do not provide any significant recharge to the phreatic aquifer.

The water extracted from the aquifer system is largely collected again by the sewage network after use. Therefore the water flow rate in the sewage network depends on: the amount of water distributed by the Water Works, which is equal to $P_a(1 - C_{al})$; P_p, the volume of water extracted by private wells in one year per unit area; C_u, the fraction of used water which is not recollected by the sewage network; C_{sl}, the fraction of water lost by the sewage network. The recharge to the aquifer due to losses from the sewage network is given by:

$$R_{sewage} = \{[P_a \cdot (1-C_{al}) + P_p](1-C_u) + R(1-C_e)C_c\}C_{sl}\, C_i \tag{5}$$

Let R_{total} denote the total recharge to the phreatic aquifer. Then

$$R_{total} = R_{rain} + R_{ww} + R_{sewage} \tag{6}$$

Table 1 lists the values of the parameters for 1974 and 1982; notice that R, C_e, C_c, C_i, C_{al}, C_u, C_{sl} have the same value for both years. Rain infiltration and losses from the sewage network are the principal contributions to the aquifer recharge. Losses from the distribution network of the Water Works are nevertheless significant. The recharge of the aquifer provides approximately a half of the total amount of water extracted in a year.

Table 1 Parameters for the determination of the aquifer recharge, aquifer recharge due to rain infiltration, losses from the Water Works and sewage buried pipes, and total recharge for 1974 and 1982.

Year	R (mm)	C_e	C_c	C_i	C_{al}	C_u	C_{sl}	P_a (m)	P_p (m)	R_{rain} (m)	R_{ww} (m)	R_{sewage} (m)	R_{total} (m)
1974	1000	0.3	0.5	0.9	0.1	0.1	0.2	1.008	1.074	0.315	0.091	0.384	0.790
1982	1000	0.3	0.5	0.9	0.1	0.1	0.2	0.901	0.297	0.315	0.081	0.242	0.638

IDENTIFICATION OF HYDRAULIC CONDUCTIVITY

The identification of the hydraulic conductivities has been conducted with a trial-and-error procedure guided by the evaluation of the average and squared differences between the model results and observations of heads for the years 1974 and 1982. For these years two classes of head data are available: piezometric heads, $\phi^{(obs)}$, measured in a well of each pumping station of the Water Works; phreatic heads, $h^{(obs)}$, measured in shallow piezometers and available from SIF (Office for the Information System on the Aquifer, Provincia di Milano). Following the remarks in the previous "Water extraction" section, we cannot attribute the measured piezometric heads to a single aquifer; on the other hand phreatic heads measured in shallow piezometers can be attributed to the phreatic aquifer with confidence.

The model computes the phreatic heads, $h_F^{(mod)}$, and the piezometric heads for the three confined aquifers, $\phi_{c1}^{(mod)}$, $\phi_{c2}^{(mod)}$ and $\phi_{c3}^{(mod)}$. We compare $h_F^{(mod)}$ with $h^{(obs)}$ and $\phi_{c1}^{(mod)}$ with $\phi^{(obs)}$. In this way we take into account the fact that the two classes of data might refer to different layers and that piezometric heads measured in the wells of the Water Works are influenced also by the heads in the confined aquifers.

There was quite a lot of confidence in the value of K_{eq} identified by Giudici *et al.* (2000); the determination of the ratio of the hydraulic conductivities of two adjacent aquifers, r, was more uncertain. Separate use of the data corresponding to the phreatic and the confined aquifers can improve the identification of r. For this reason a small interval of variation for K_{eq} and a wider range for r are considered in this paper.

The choice of the optimal values is guided by the evaluation of the squared errors, i.e. the functions:

$$J_F(K_{eq},r;t) = \left(\frac{1}{N}\sum\left(h^{(obs)} - h_F^{(mod)}\right)^2\right)^{1/2} \quad \text{and}$$

$$J_C(K_{eq},r;t) = \left(\frac{1}{N}\sum\left(\phi^{(obs)} - \phi_{c1}^{(mod)}\right)^2\right)^{1/2} \tag{7}$$

where the parameter t denotes the considered year (i.e. 1974 or 1982) and the sum is extended to all the points where a measurement is available for the corresponding aquifer; the number of measurement points is N.

In Fig. 2 the functions $J_F(K_{eq},r;t)$ and $J_C(K_{eq},r;t)$, for t = 1974,1982, as computed with the wells pumping from both the phreatic and the confined aquifers, are represented as contour plots. The functions $J_F(K_{eq},r;t)$ and $J_C(K_{eq},r;t)$ are characterized by opposite trends. The variations of $J_C(K_{eq},r;t)$ are sharper than those of $J_F(K_{eq},r;t)$, in particular as r increases and for small values of K_{eq}. Furthermore the variations are sharper for t = 1974 than for t = 1982, because in the former case the source terms are greater and produce higher sensitivity of the piezometric heads to the hydraulic conductivity. As mentioned before, a small interval of variation for K_{eq} and a wider range for r are considered; for this reason the functions do not appear as ellipsoids with a definite minimum point.

The results obtained assuming that the wells extract water from the confined aquifers only are very similar to those of Fig. 2, both from the qualitative and quantitative points of view. Therefore these results do not permit distinction between the two alternatives, i.e. they do not allow us to state without any doubts whether pumping takes place through the whole thickness of the aquifer system or only in the confined aquifers.

The cumulative analysis of $J_F(K_{eq},r;t)$ and $J_C(K_{eq},r;t)$ for the two years allows us to improve the optimal estimate of r, to which the value $r = 3$ is now assigned, and to

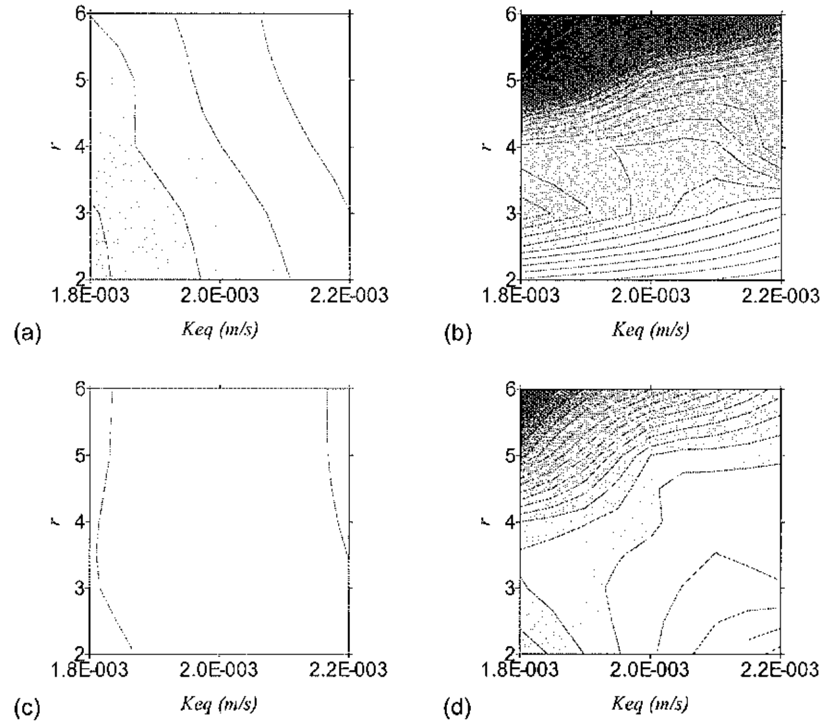

Fig. 2 Contour plots of (a) $J_F(K_{eq},r;\ 1974)$, (b) $J_C(K_{eq},r;\ 1974)$, (c) $J_F(K_{eq},r;\ 1982)$ and (d) $J_C(K_{eq},r;\ 1974)$. Equidistance: 0.2 m. Colour scale ranges from 3.4 m (white) to 9.6 m (black).

confirm the value of $K_{eq} = 2 \times 10^{-3}$ m s^{-1}, previously identified by Giudici *et al.* (2000). In particular, the new value of r reduces the difference between the modelled and the measured heads in the confined aquifers.

SENSITIVITY WITH RESPECT TO SOURCE TERMS

We perform a sensitivity analysis to evaluate the effects on the model results produced by errors in the estimates of the parameters that determine the aquifer recharge. Let x denote any parameter (i.e. R, P_a, P_p, C_e, C_c, C_{al}, C_u, C_{sl}, C_i), which is considered as independent variable; let y denote any recharge term (i.e. R_{rain}, R_{ww}, R_{sewage}, R_{total}), which is considered as dependent variable. The relative variation of y, which corresponds to a unit relative variation of x, is computed as:

$$\mathcal{S}(y,x) = \frac{x}{y}\frac{\partial y}{\partial x} \tag{8}$$

We refer to $\mathcal{S}(y,x)$ as the sensitivity of the recharge term y to the parameter x. The sensitivities are easily computed from equations (3) to (6). The sensitivities of the total recharge, R_{total}, computed for both years 1974 and 1982 are listed in Table 2.

A key parameter for correct determination of aquifer recharge is C_i, because the relative error on this parameter yields a unit relative error in the total recharge. Notice

Table 2 Sensitivity coefficients of R_{total} with respect to the coefficients that parameterize the aquifer recharge for the years 1974 and 1982.

Year	R	P_a	P_p	C_e	C_c	C_u	C_{al}	C_{sl}	C_i
1974	0.48	0.30	0.22	−0.21	−0.32	−0.05	0.09	0.49	1.00
1982	0.59	0.33	0.08	−0.25	−0.39	−0.03	0.10	0.38	1.00

that C_{sl}, C_c and C_e are also very important; their estimates are presently very uncertain. In particular the best value of C_{sl} could be lower than the value which has been assigned. The parameters R, P_a and P_p are known with good accuracy. The lowest values of sensitivity are obtained for the parameters C_u and C_{al}.

CONCLUSIONS

This paper describes improvements to the calibration of the model of groundwater flow in the aquifer system located beneath the city of Milan (Italy). Different datasets have been used, i.e. measurements of phreatic and piezometric heads corresponding to different flow conditions; this is a very powerful means to obtain more reliable estimates of the model parameters, as shown with theoretical or numerical results by Scarascia & Ponzini (1972), Parravicini *et al.* (1995), Giudici *et al.* (1995), Snodgrass & Kitanidis (1998).

The parameterization of the source terms has been reviewed and a sensitivity analysis shows the necessity of further studies of some processes, e.g. water infiltration through the unsaturated zone, the characteristics of the sewage network (fraction of rain collected by the sewage, loss coefficient of the sewage system).

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