Calibration and Reliability in Groundwater Modelling (Proceedings of the ModelCARE 99 Conference held at Zürich, Switzerland, September 1999). IAHS Publ. no. 265, 2000.

Coupling groundwater models to external simulation modules: strategies and application to surface waters

C. KAISER, H.-J. G. DIERSCH, R. GRÜNDLER
WASY Institute for Water Resources Planning and Systems Research Ltd, Waltersdorfer Strasse 105, D-12526 Berlin, Germany
e-mail: kristian@wasy.de

V. ZLOTNIK
Department of Geoscience, University of Nebraska-Lincoln, 214 Bessey Hall, Lincoln, Nebraska 68588-0340, USA

Abstract We investigate the hydraulic interaction of a groundwater resource with a lake when abandoned open-cast mines are flooded. We discuss two approaches in modelling the combined system. When using finite element simulators the holistic approach provides a direct treatment in a single matrix system. The decomposed approach handles the subprocesses separately and connects them via boundary conditions. We emphasise the importance of embedding the lake in a groundwater body of appropriate spatial dimensions. By studying the representation of a lake in one-dimensional, two-dimensional (2-D) vertical and horizontal, and three-dimensional (3-D) confined flow models we assess the systematic error of the lake's filling dynamics. We show that for long-term predictions of the water table rise in a mine the error becomes intolerable in 2-D regional models. To fully account for the filling behaviour of a lake its volume must be carved out of a 3-D groundwater model with an accurate surface geometry.

INTRODUCTION

The management of water resources on a catchment scale is only feasible by harnessing the predictive power of computer simulation models for the assessment of management scenarios. Therefore, hydrologists have developed a growing need for integrated computer models which are able to treat separate hydraulic flow problems in one go. Typical applications in practical modelling concern the interaction of the groundwater resource with surface waters like rivers and lakes. We focus here on the water exchange between aquifers and lakes in full hydraulic contact. This problem occurs in abandoned lignite mines when they are flooded for re-use as reservoirs to regulate the regional water balance. Large modelling projects have been undertaken to predict the long-term filling behaviour of the mines. Interaction processes are often taken into account in a heuristic manner without assessing possible systematic errors arising from the coupling technique or the approximation of complex lake geometries. In this paper we explore the possible coupling approaches systematically by investigating a one-dimensional (1-D) model based on an analytical solution, two-dimensional (2-D) vertical and horizontal models, and three-dimensional (3-D) regional models. The models are set up with the finite-element groundwater simulator FEFLOW (Diersch, 1998).

Both the holistic and the decomposed approach are commonly used. The holistic strategy is the most rigorous where all processes to be modelled are handled directly. In the finite element context a single matrix system contains all the information about the groundwater-surface water model. A lake can be represented in the groundwater model as an unconfined aquifer with high hydraulic conductivities and a storage coefficient equal to unity. Furthermore, mixed elemental units with varying spatial dimensions and physical characteristics can be introduced such as 1-D channels, 2-D overland flow and 3-D subsurface elements with the fluid motion governed by different laws. In the decomposed approach the overall system remains partitioned into the natural sub-processes with their own temporal and spatial units. They are linked together with appropriate boundary conditions. Mathematically this is termed an embedded procedure. Once the solution for the first system is known its boundary values are put in to the second for an independent treatment. Most of the results presented here have been calculated using the decomposed approach.

The software realization within the FEFLOW package is facilitated by the Interface Manager (IFM) (Gründler *et al.*, 1998). The IFM provides access to FEFLOW's internal simulation procedures via well-defined call-back functions. Typical functions are PreTimeStep() and PostTimeStep() where the material parameters and boundary conditions can be manipulated before and after a time step is performed. The IFM is based on the concepts of Computer Aided Software Engineering (CASE) and allows users to couple their own external simulation modules to FEFLOW.

ONE-DIMENSIONAL MODEL

The only way to verify both the holistic and the decomposed approaches is to compare them to an independent analytical solution which can only be obtained for a confined aquifer in 1-D as shown in Fig. 1. The filling curve of the lake is determined by two parameters: the lake width w and the embankment angle α. The relation between the lake water table h and the volume V is:

$$h(V) = \frac{-w + \sqrt{w^2 + 4\tan(\alpha)V}}{2\tan(\alpha)} \tag{1}$$

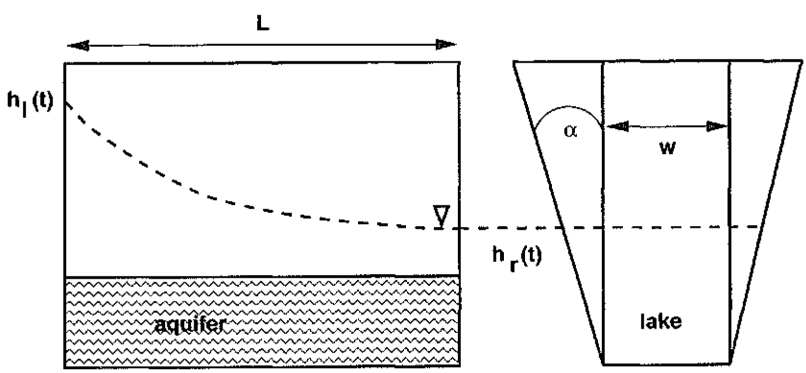

Fig. 1 Schematization of the aquifer–lake system.

Analytical solution

We consider a 1-D confined aquifer with transmissivity T and storage compressibility S of length L. Inside the aquifer the hydraulic head is derived from the equation of motion:

$$S\frac{\partial h(x,t)}{\partial t}=T\frac{\partial^2 h(x,t)}{\partial x^2} \tag{2}$$

The lake of width w and with a vertical embankment is hydraulically connected to the aquifer with a mass balance:

$$w\frac{\partial h_r(t)}{\partial t}=-T\frac{\partial h(L,t)}{\partial x} \tag{3}$$

for the water flow between aquifer and lake. In equation (2) we have used the continuity of the hydraulic head $h_r(t) = h(L,t)$ at the right aquifer boundary. We now impose an arbitrary dynamic head:

$$h(0,t) = h_l(t) \tag{4}$$

on the left boundary. The head inside the aquifer and in the lake was initially constant at $h(x,0) = h_0$ for $0 \le x \le L + w$. After applying a Laplace transformation to equations (2)–(4) and accounting for the boundary conditions we obtain the end result:

$$h_r(t)=\mathcal{L}^{-1}\left[\frac{\bar{h}_l(p)}{\cosh(\sigma(p)L)+\sqrt{\frac{w^2p}{ST}}\sinh(\sigma(p)L)}\right] \quad \text{with} \quad \sigma(p)=\sqrt{\frac{S}{T}p} \tag{5}$$

where $\bar{h}_l(p)$ denotes the transformed left boundary condition and $\mathcal{L}^{-1}$ denotes the inverse Laplace transformation with respect to the Laplace parameter p.

Numerical calculations

For the 1-D model the aquifer length was L =100 m, the lake width was set to w = 5 m and the embankment angle α was zero. The aquifer transmissivity was set to $T = 10^{-4}$ m s^{-1} and the storage compressibility $S = 10^{-4}$. We started with an initial water table of h_0 = 1 m and the left boundary table $h_l(t)$ went up linearly from 1 m to 41 m in 20 days (Fig. 2). To obtain the analytical results the back-transformation of equation (5) was performed with the Stehfest algorithm (Stehfest, 1970). In the decomposed approach the mass balance at the right boundary is implemented by using the IFM. A small IFM module retrieves the water table $h_r(t)$ and the flux $q_r(t)$ between the aquifer and the lake at a given time step t and computes the change $\Delta h_r(t) = \Delta t/w\ q_r(t)$ of the hydraulic head between subsequent time steps for a given filling curve of equation (1). This change is added to the actual boundary head: $h_r(t) + \Delta h_r$ is set to $h_r(t)$ and the finite element problem is solved with the new head. A time step of Δt = 0.1 day and a mesh of 100 equally distanced finite elements were found to yield sufficient accuracy.

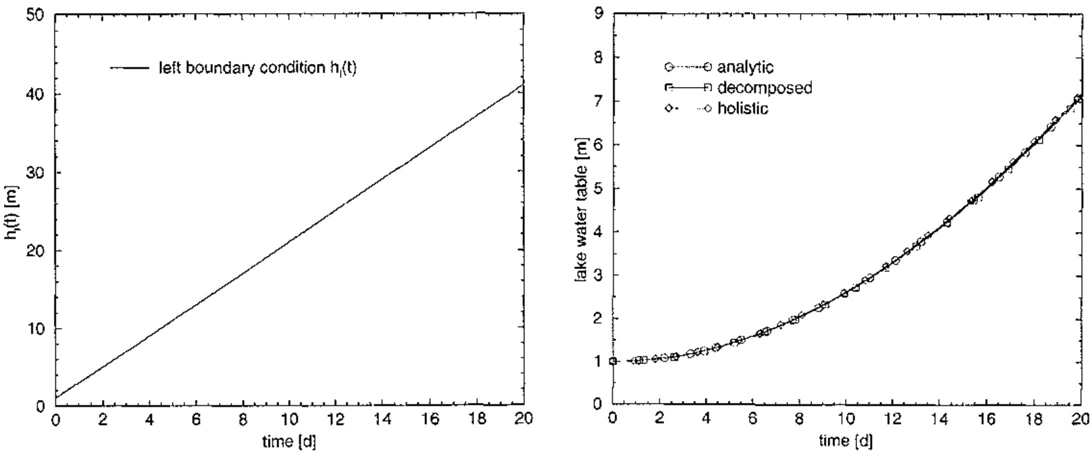

Fig. 2 Left boundary excitation (left) and response of the lake water table computed with the holistic, decomposed and analytical approaches (right).

To model a confined aquifer with the holistic approach the top elevation and the aquifer thickness were set to 1 m. For the lake an unconfined aquifer with a width of $w = 5$ m, a hydraulic conductivity of $k_f = 1$ m s^{-1} and a storage coefficient of $S = 1$ was added to the model at the right end. Figure 2 shows that both approaches match the analytical result with sufficient accuracy. After 20 days, the error is 5 cm for the holistic and 2.5 cm for the decomposed calculations, respectively.

REPRESENTING THE LAKE'S GEOMETRY

In the previous 1-D model the embankment of the lake was vertical and no geometry effects entered the calculations. But when Rembe & Wenske (1998) proposed to use the lake boundary condition of the decomposed approach in 2-D regional groundwater models, the interface for water exchange between the aquifer and the lake was no longer represented correctly. The depth integrated equation of water motion produces a vertical aquifer interface whereas the lake interface has a slope for embankment angles $\alpha > 0$. This geometry mismatch is shown in Fig. 1. We have checked the systematic error by comparing a 1-D model with a 2-D vertical model that possesses the true aquifer–lake interface. The angle α is set to 80° and the lake width is 1 m. The 1-D aquifer is 400 m long, the top length of the 2-D vertical model is 375 m and the base length is 425 m, with a height of 50 m. The left boundary head rose from 51 m to 91 m in 20 days.

Figure 3 shows the head distribution after 20 days. For the 1-D model the expected constant head gradient can be compared with the distorted head distribution of the vertical model. The distortion is due to the slope of the lake embankment. The Darcy flux into the lake in the 1-D model is 0.84 m day^{-1}. In the 2-D model large head gradients appear at the top edge of the lake and induce a maximum flux of 2.8 m day^{-1} which decays exponentially to zero at the bottom of the lake. The integrated horizontal Darcy flux along the interface is larger than in the 1-D model. Thus, the lake is filled faster than in the 1-D case as shown in Fig. 4. The difference increases with increasing

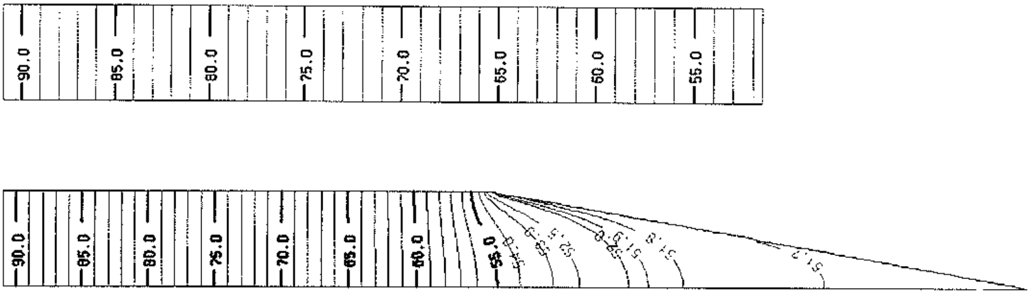

Fig. 3 Head distribution (in m) after 20 days for the 1-D (top) and 2-D vertical (bottom) model; lake with embankment angle $\alpha = 80°$ (schematic sketch without scale, for true measurements see text).

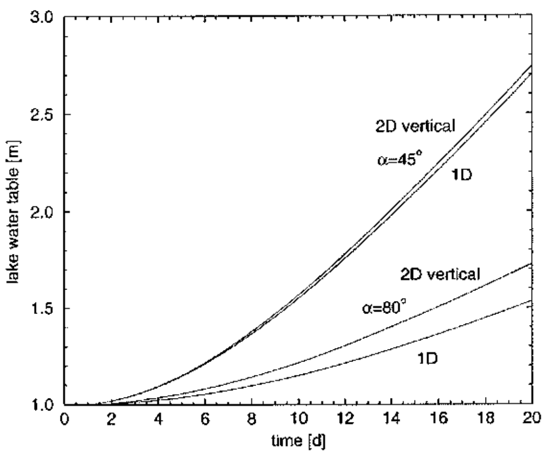

Fig. 4 Lake water table for the 1-D and 2-D vertical model for $\alpha = 45°, 80°$ (measured from the aquifer top at 50 m).

angle α and is already around 25 cm for $\alpha = 80°$. It is almost halved if the aquifer length is doubled and one may expect it to be less pronounced in regional models.

Regional models

Having demonstrated the effect of the geometry mismatch in principle we will show how it determines the filling dynamics in regional models. We have set up two confined 3-D models with squared base areas of edge lengths 1500 m and 10 000 m with a lake ($\alpha = 80°$, $w = 150$ m) carved out in the upper right corner. They are 50 m thick and have been discretized with five layers and approximately 1000 elements per layer. The dynamic head boundaries were applied to both edges opposite to the lake.

For the smaller model we assumed the same time dependence as in the 2-D vertical model. Figure 5 shows the flow field of the top layer after 20 days compared to the corresponding 2-D model. Whereas the flow patterns far away from the lake do not differ much, the flow is drastically altered in the lake's immediate vicinity. The Darcy

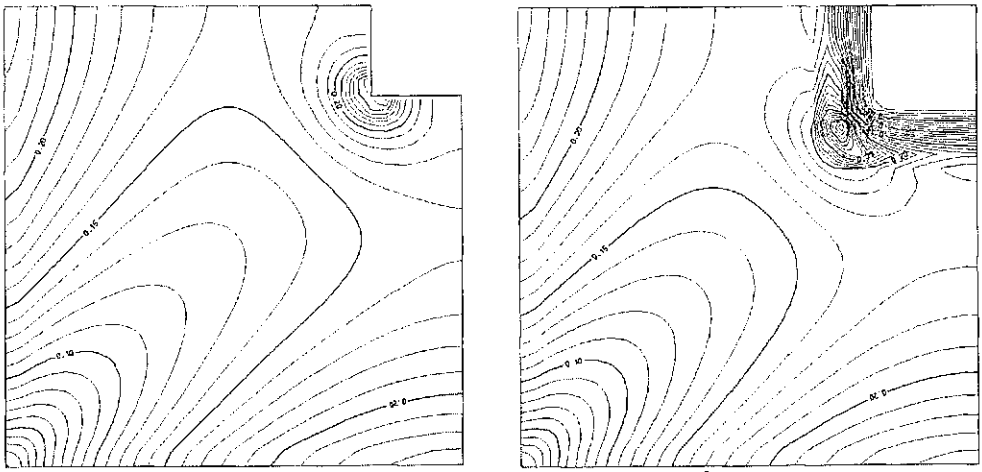

Fig. 5 Absolute values of Darcy velocities (in m day^{-1}) after 20 days in the top layer of the 2-D horizontal (left), and 3-D (right) model with edge length 1500 m, lake with angle α = 80°, w = 150 m. Thick line labels: 0.1 m day^{-1} in the lower left corner, and from there increasing to the upper right corner with an interval of 0.05 m day^{-1}.

fluxes are higher in the top layer of the 3-D model compared to the 2-D model. The 3-D maximum flux is 0.34 m day^{-1} whereas the 2-D flux is 0.26 m day^{-1}. As in the 2-D vertical case, the lake is filled from above in the 3-D case with faster filling dynamics. The water table difference between the 2-D and the 3-D model after only 20 days is 2 m. For the large regional model we assumed a slow water table rise of 10 m year^{-1}. Now the water table difference increases grows linearly with a rate of 3 cm year^{-1}. After five years, when the lake is filled, it is already 15 cm.

CONCLUSION

For large regional groundwater models the systematic error in the rise of the water table of an embedded lake increases linearly when 2-D horizontal and 3-D models are compared. The growth rate of the error depends on the size of the model and the lake for a given rise rate of the surrounding groundwater level. We have computed typical values of 1–10 cm year^{-1}. Long-term assessment studies for open-cast mines can cover more than ten years. The accumulated error then reaches a difference of 0.1–1 m which is intolerable when compared to typical water table differences of 1–2 m which are operationally used in a lake reservoir to regulate the regional water balance. Furthermore, the 2-D model always underestimates the filling dynamics. Hence, if one intends to apply the decomposed approach in the studies, we recommend use of 3-D models and that the lake's surface geometry is carved out of the groundwater body as accurately as possible.

Acknowledgement The authors thank Steffen Bold for doing part of the FEFLOW calculations.

REFERENCES

Diersch, H.-J. G. (1998) *Interactive, Graphics-based Finite Element Simulation System FEFLOW for Modelling Groundwater Flow, Contaminant Mass and Heat Transport Processes, User's Manual (release 4.7).* WASY Ltd, Berlin, Germany.

Gründler, R., Kaiser, C., Diersch, H.-J. G. & Voigt, R. (1998) An open programming interface for groundwater modelling systems. In: *Hydroinformatics 98* (Copenhagen, Denmark), 597–604. ISBN 9-054109-83-1.

Rembe, D. & Wenske, D. (1998) Die Seerandbedingung—eine zusätzliche Randbedingung für das modulare dreidimensionale Grundwasserströmungsmodell MODFLOW. *Mathematische Geologie* **2**, 45–56.

Stehfest, H. (1970) Numerical inversion of Laplace transforms. *Commun. ACM* **13**(1), 47–49.