

## Multiphase modelling of gas storage in an aquifer with automatic calibration and confidence limits

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**Abstract** Multiphase flow modelling involving gas and water is widely used in gas dissolution in aquifers or in aquifer gas storage. The parameters related to the gas are usually well known but the parameters of the aquifer system are not. In order to obtain reliable forecasts, it is necessary to calibrate the multiphase model on monitored data. This can be done by automatic calibration followed by the determination of the confidence limits of the parameters, and of the confidence limits of the forecasts. This paper presents a three-dimensional multiphase flow model for the simulation of gas–water, or NAPL–water couples. It integrates automatic calibration procedures based on Marquardt’s algorithm for all types of parameters: hydraulic conductivity, which is classical, and also multiphase flow constitutive relations, aquifer compressibility and boundary limits. The model may be calibrated simultaneously on observations of different kinds of data, e.g. gas pressure, hydraulic heads, water saturation. A sensitivity analysis method, which integrates spatial or temporal dependence of the observations, follows the automatic calibration. The model is applied to data from a gas storage facility located in a gas cap in an aquifer near Paris (France); it has been monitored by the national Gaz de France company for more than 30 years. The automatic calibration enabled determination of the hydraulic conductivity, the rock compressibility and the lateral boundary limits. It resulted in a very accurate simulation of the gas pressure changes resulting from the periodic injections and withdrawals of gas. The sensitivity analysis enabled determination of the parameters that were the most important.

### NOTATION

$A$	exchange area between cells [ $L^2$ ]	$Vol$	volume of a cell [ $L^3$ ]
$hc$	suction = $hg - hw$ [L]	$hg$	gas hydraulic pressure head [L]
$hw$	water hydraulic pressure head [L]	$H_w$	water hydraulic head = $hw + z$ [L]
$STO_g$	stored mass of gas [M]	$STO_w$	stored volume of water [ $L^3$ ]
$x, dx$	distance between two cells [L]	$t, dt$	time, time step [T]
$z$	elevation [L]	$k$	permeability [ $L^2$ ]
$K_{rw}$	water relative conductivity [–]	$K_{rg}$	gas relative conductivity [–]
$\beta_w$	water compressibility [ $L^{-1}$ ]	$\beta_{por}$	pore compressibility [ $L^{-1}$ ]
$\theta_g$	volumetric gas content [ $L^3 L^{-3}$ ]	$\theta_w$	volumetric water content [ $L^3 L^{-3}$ ]
$\mu_g$	gas viscosity [ $M L^{-1} T^{-1}$ ]	$\mu_w$	water viscosity [ $M L^{-1} T^{-1}$ ]

$\rho_g$	gas density [M L <sup>-3</sup> ]	$\rho_w$	water density [M L <sup>-3</sup> ]
$\omega$	porosity = $\theta_w + \theta_g$ [L <sup>3</sup> L <sup>-3</sup> ]	$d_g$	gas relative density = $\rho_g/\rho_w$ [-]
$Q_w$	volumetric flow of water [L <sup>3</sup> T <sup>-1</sup> ]	$Q_g$	mass flow of gas [M T <sup>-1</sup> ]
$g$	gravity acceleration [L T <sup>-2</sup> ]		

## INTRODUCTION

Depending on the season, the consumption of natural gas undergoes significant fluctuations. The fact that supply sources tend to be remote complicates the task of adapting deliveries to seasonal and daily variations in demand. Thus as far back as 1954, Gaz de France began developing techniques for storing natural gas underground. Underground storage facilities help balance the sharp contrasts in demand between winter and summer. They also serve to meet peaks in demand on the coldest winter days. Moreover, they have to be ready to deal with exceptional situations like the record cold spells that occur statistically twice in every century. Last, they compensate for the possible failure of a supply source. The gas storage inventory reflects the evolution of the demand for gas that has risen steadily, especially in winter.

To meet this demand, it is necessary to be focused both on the quality of the data, in order to have a very precise knowledge of the storage at all times, and on the quality of the reservoir modelling tool that simulates the storage behaviour and forecasts its future performance. For the forecasts to be valid, a numerical simulation should first adequately reproduce the past performance of the reservoir. The task of finding a reservoir description which correctly simulates the past performance of the reservoir is called history matching (HM). For the HM to be correct, we need to have accurate measured pressures, an appropriate reservoir modelling tool and a suitable HM technique using automatic calibration. The difference between simulated outputs and measured data, i.e. the data residuals, will then be acceptably small, and the performance forecasts reliable.

## MULTIPHASE MODELLING OF FLOWS

### Equations to be solved

At each time step it is necessary to solve a system of nonlinear equations resulting from: (a) the volume conservation of water in each cell, (b) the mass conservation of gas in each cell, and (c) the equations of state for gas and water. These equations, expressed in finite volume, are:

Volume conservation of water (subscript w):

$$\sum_i T_{wi} (H_i - H_w) + Q_w = \frac{d(STO_w)}{dt} \tag{1}$$

Mass conservation of gas (subscript g):

$$\sum_i T_{gi} \rho_{gi} [(h_{gi} - h_g) + d_{gi} (z_i - z)] + Q_g = \frac{d(STO_g)}{dt} \tag{2}$$

with:

$p$	index for the beginning of the time step
$i$	index of an adjacent cell
$\rho_{gi}$	weighted gas density with adjacent cell $i$
$d_{gi}$	average relative density with cell $i$
$T_{wi}$	$k \rho_w / \mu_w K_{rw} A / dx$
$T_{gi}$	$k (\rho_w / \mu_g) K_{rg} A / dx$
$STO_w$	$Vol [(\theta_w - \theta_{wp}) + \theta_w (\beta_w + \beta_{por})(H_w - H_{wp})]$
$STO_g$	$Vol [(\theta_g \rho_g - \theta_{gp} \rho_{gp}) + \rho_g \theta_g \beta_{por} (h_g - h_{gp})]$

### Resolution of the system of equations

The equations are solved with the three-dimensional (3-D) finite volume code MARTHE described by Thiéry (1990, 1993b). Three formulations are available for the unknowns: (water head – volumetric content) or (gas pressure – volumetric content) or (water head – gas pressure). Picard iterations are used to take care of the nonlinearities. The transfer coefficient between two adjacent cells uses an upstream weighting of the adjacent relative conductivities (i.e. it is the relative conductivity of the upstream cell which is used). This scheme has proved to be very stable and efficient. For the present case a “water head – gas content” scheme has been chosen. In each cell the unknowns are the head of the water phase,  $H_w$ , and the gas content,  $\theta_g$ . The volume and mass conservation equations are written with these variables converting  $\theta_w$  to  $\theta_g$  using the relation  $\omega = \theta_w + \theta_g$  and converting  $hg$  to  $H_w$  using the definition of suction,  $h_c$ . Then  $\theta_g$  is eliminated by adding equation (2) to equation (1) multiplied by  $\rho_g$ . One obtains a system of  $n$  equations,  $n$  being the number of cells, with the only variable  $H_w$  which is solved classically. From the computed values of  $H_w$ , the variables  $\theta_w$  are immediately obtained explicitly in each cell from equation (1). Then in each cell  $hg$  is obtained immediately from the suction definition, and  $\theta_g$  from  $\omega = \theta_w + \theta_g$ .

### Geometry of the model, boundary conditions and initial parameters

The underground gas storage of Saint-Illiers-la-Ville is located 50 km west of Paris (France) in the Upper Oxfordian. It is a fairly radial sandstone reservoir, 30 m thick, situated 340 m below sea level. The aquifer gas storage now forms a central gas cap surrounded by an active aquifer. The maximum content is about 1500 million cubic metres ( $1500 \times 10^6 \text{ m}^3$ ) at normal conditions, and the working gas volume is roughly  $650 \times 10^6 \text{ m}^3$ . Figure 1 shows a plan view of the reservoir.

The reservoir has been modelled in radial coordinates. The grid is as follow: a circular cell of radius 500 m, where gas injection or withdrawal take place, surrounded by 22 ring cells 50 m wide. Then larger ring cells extend the domain to a radius of 82 km (Ory *et al.*, 1997). The vertical thickness of each ring cell is 30 m and the top of the formation decreases from –340 m (below m.s.l.) at the centre to –460 m at a distance of 1.7 km. In each cell the initial value of permeability  $k$  has been fixed to 2 Darcys ( $1.97 \times 10^{-12} \text{ m}^2$ ) and the initial value of the porosity  $\omega$  to 28%. The

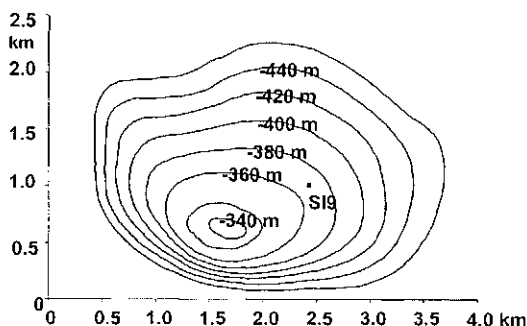


Fig. 1 Plan view of Saint-Illiers reservoir: isobathes and the observation well. The production wells (not shown) are located at the top of the reservoir.

compressibility of the pores,  $\beta_{por}$ , has been fixed to  $2 \times 10^{-5} \text{ m}^{-1}$  and the compressibility of the water,  $\beta_w$ , to  $0.5 \times 10^{-5} \text{ m}^{-1}$ . The constitutive laws for the relative permeability, the capillary pressure (suction), the correction of the gas compressibility with the pressure (the natural gas is not a “perfect gas”) and the gas viscosity have been provided by Gaz de France. The gas pressure is monitored at the head of the observation well SI9 that corresponds approximately to the second cell of the model (525 m from the injection). The pressure history is corrected to a reference datum of 360 m below sea level.

The rate of gas injection or withdrawal and the gas pressure measurements are available for the 33 year period extending from 1965 (when natural gas injections started) to 1997. 1656 pressure measurements are available which corresponds to weekly observations. At the beginning of the storage (1965) there was no gas and the water phase was immobile with a hydraulic head estimated at 141.11 m which corresponds to a pressure of 616.11 m (60.44 bars) at the outer limit where the mean altitude is  $-475 \text{ m}$ . At the outer limit the hydraulic head is prescribed and the gas content is prescribed at 0. Considering the uncertainty of the hydraulic head at the outer limit, and the relative altitude where the gas pressure is monitored, a correction has been applied to this head with an initial value  $H_0 = +30 \text{ m}$  (+2.94 bars). The simulation with MARTHE code has been performed with automatic time steps prescribed to range between 0.5 and 4 days. On a classical PC computer (Pentium II) approximately 6 min of CPU time are needed for a simulation of the total period.

## AUTOMATIC CALIBRATION OF THE PARAMETERS

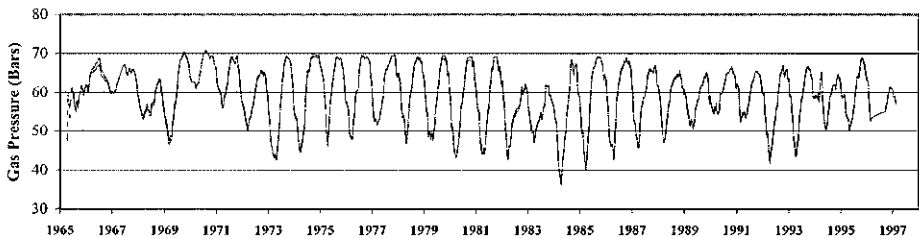
The Marquardt algorithm (Marquardt, 1963; Press *et al.*, 1992), which has been implemented in MARTHE code, has been used to calibrate four parameters:  $k$ ,  $\omega$ ,  $\beta_{por}$  and  $H_0$ . This algorithm, which is more efficient than the former method described by Thiéry (1993a, 1994), necessitates the computation of the gradients of the observations (the monitored pressure) with respect to each parameter in order to get a Hessian. The gradients have been computed by finite difference, which necessitates, for each iteration of the algorithm, one simulation per parameter plus a reference simulation. Anterion *et al.* (1989) present a faster method for the direct calculation of all the gradients in a single simulation which could be used.

Table 1 displays the evolution of the calibration of the four parameters. The standard deviation of the observations  $\sigma_{obs}$  is 72.05 m (7.07 bars) and the initial root mean square of error,  $RMSE$ , is 25.83 m (2.53 bars). The calibration criterion is defined as  $(RMSE/\sigma_{obs})^2$  which is the percentage of residual errors (unexplained variance). After two iterations the criterion is reduced from 12.8% to 0.96%. After four iterations the parameters are calibrated and the  $RMSE$  is equal to 5.99 m (0.6 bar).

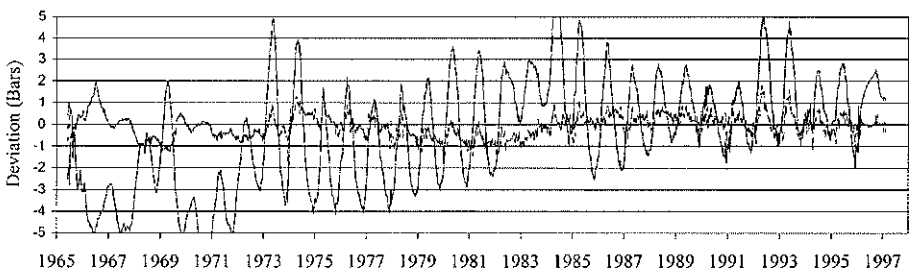
**Table 1** Automatic calibration of parameters.

Iteration:	0	1	2	3	4
Permeability (D)	2	0.519	0.669	0.688	0.729
Porosity (%)	28	31.3	29.6	24.5	22.45
Head correction (m)	30	21.2	10.4	-2.78	-7.89
Pore Compressibility ( $m^{-1}$ )	2	7.82	2.68	1.36	0.857
Criterion (%)	12.8	3.60	0.963	0.713	0.693
RMSE (m)	25.83	13.66	7.07	6.09	5.99
RMSE (bars)	2.53	1.34	0.69	0.597	0.588

Figure 2 displays the comparison of the monitored gas pressures to the simulated ones. As the simulation with the MARTHE code is very accurate the two curves can hardly be separated on the graph. It should be noted that the low pressures due to a strong depletion in the winter of 1984–1985 and the peak resulting from significant increases in maximum storage in the summer of 1995 are well simulated. Figure 3 displays the initial and final series of the pressure deviations.



**Fig. 2** Monitored and simulated gas pressure at the observation well.



**Fig. 3** Initial (large amplitude) and final (small amplitude) series of pressure deviations.

### SENSITIVITY ANALYSIS

The sensitivity analysis is based on a local linearization of the model in the vicinity of the optimal set of parameters. The method, which uses also the Hessian matrix, has been described among others by Thiéry (1993b,c) and Leijnse (1982). Classically the method is applied assuming that the observations, or the simulation residuals, are independent which is not acceptable most of the time. Instead Thiéry (1993b,c) shows that it is necessary to integrate their spatial or temporal dependence. This could be done using the covariance matrix of observations. However this is not feasible given the large number of observations which would yield a huge matrix. An approximate method has been used instead. The analysis of the monitored pressures shows that two pressures separated by 10 time lags (approximately 70 days) have an autocorrelation coefficient of not less than 0.5. It can then be considered that roughly a set of 10 consecutive observations forms only one independent observation. The total number of independent observations reduces from 1656 to 166, which is realistic because it corresponds to 5.2 independent observations per year over 31.8 years. Using this method the standard deviation of the parameters has been calculated. Three parameters,  $k$ ,  $\omega$  and  $\beta_{por}$  had a logarithmic transformation. For these parameters relative standard deviation is displayed. The relative standard deviation is based on the standard deviation of the logarithm. It defines the range which has a probability of 68.2% of containing the true value. Table 2 displays the correlation matrix of the parameters and the standard deviations. It appears that two parameters are precisely defined:  $k$  and  $\omega$  (standard deviation 3–5%). Two parameters are strongly correlated:  $k$  and  $\beta_{por}$  (correlation  $-0.923$ ). The negative sign indicates that a large compressibility has a similar effect to a large permeability: both parameters tend to dampen pressure variations.

Table 2 Sensitivity analysis of the parameters.

	Permeability	Porosity	Head correction	Compressibility	Optimal value	Standard deviation	Relative standard deviation
Permeability	1				0.729		4.89%
Porosity	0.064	1			22.45		2.74%
Head correction	0.619	0.791	1		-7.89	2.55 m	
Compressibility	-0.923*	0.054	-0.462	1	0.857		34.5%

The sensitivity analysis shows that the rock compressibility is not very well identified. This parameter could be fixed to a reference value. The other parameters would then be better identified.

### CONCLUSIONS

The multiphase flow numerical method used in MARTHE code performs well and the automatic calibration scheme is efficient even in highly non linear systems. It enables very accurate simulation of the variations of gas pressure in a storage reservoir over a long monitored period which proves its ability to yield reliable predictions. The sensitivity analysis presented is useful in determining the most significant parameters.

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