A comparison of different model concepts for saltwater intrusion processes

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Abstract In this paper a one-phase/two-component and a two-phase flow model concept for simulating saltwater intrusion processes are investigated. For both, the governing equations as well as the discretization and solver techniques, which are included in the numerical simulator MUFTE-UG, are briefly explained. Both model concepts are compared using a single-layered and a two-layered system as well as the corresponding semi-analytical solutions of Henry (1964), and Mualem & Bear (1974). A very good agreement of analytical and numerical results has been obtained. As density driven flows are very sensitive, the choice of the model concept has an important influence on the results. Dispersion mainly determines the shape as well as the position of the freshwater/saltwater transition zone.

INTRODUCTION

Groundwater accounts for almost half of the water supply for human consumption. In the coastal environment, excessive groundwater pumping has caused severe intrusion of saline water into freshwater aquifers in most parts of the world. In addition, it was discovered at several locations in the North and Baltic Sea that contaminant transport into the freshwater/saltwater transition zone is caused by submarine groundwater discharge, which has a significant influence on the surface water quality. Since monitoring of such processes only provides the present state, numerical simulation has become very important as it is able to predict the impact of future trends as well as of remediation measures.

In the field of modelling saltwater processes, two different approaches are distinguished. The first assumes that fresh and saltwater are different phases which are not soluble in each other. Consequently, these two liquids are separated by a sharp interface, and a two-phase flow model concept must be applied (Essaid, 1990; Huyakorn *et al.*, 1996; Sheta *et al.*, 1998). This simplification is quite reasonable when the transition zone is narrow. The second approach takes into account that fresh and saltwater are miscible. Thus a mixing zone occurs where the salinity gradually changes due to hydrodynamic dispersion, and a one-phase/two-component model concept is required (Oldenburg & Pruess, 1995; Ataie *et al.*, 1997; Kolditz *et al.*, 1998). The second approach must be chosen when the width of the transition zone, i.e. the hydrodynamic dispersion, cannot be neglected.

In this paper, a comparison of the two different model concepts for simulating saltwater intrusion processes is presented. To achieve this, the numerical algorithms, which are part of the simulator MUFTE-UG are explained, and a single-layered as well as a multilayered system are analysed using semi-analytical solutions and sensitivity analyses.

NUMERICAL ALGORITHM

Governing equations for the two-phase flow model concept

The balance equations for the flow of two immiscible fluid phases in porous media are given by the conservation of mass (equation (1)) and the generalized Darcy law (equation (2)). The phases are called the wetting (w) and non-wetting phase (n), where w and n correspond to freshwater f and saltwater s, respectively.

$$\frac{\partial(\phi\rho_\alpha S_\alpha)}{\partial t}+\nabla\cdot(\rho_\alpha v_\alpha)=\rho_\alpha q_\alpha \qquad \text{in } \Omega xI,\ \alpha=f,s \tag{1}$$

$$v_\alpha=-\frac{k_{r\alpha}}{\mu}K(\nabla p_\alpha-\ \rho_\alpha g) \qquad \alpha=f,s \tag{2}$$

In these equations ϕ denotes porosity, ρ density, S unknown saturation, t time, v volumetric flux vector, q source or sink term, Ω space dimension, I time dimension, k_r relative permeability, μ dynamic viscosity, K absolute permeability tensor, p unknown pressure and g gravity.

In addition, two algebraic relations close the system: The void space in the porous medium is completely filled (equation (3)), and the difference of the pressure at every point is a function of the capillary pressure p_c (equation (4)):

$$S_f+S_s=1 \tag{3}$$

$$p_s-p_f=p_c \tag{4}$$

Inserting equation (1) into equation (2) and using the relations (3) and (4) leads to the pressure/saturation formulation. The choice of the primary variables is determined by appropriate boundary conditions. For the solution of these two coupled nonlinear partial differential equations, initial and boundary conditions must be given. There exist different functions to describe the constitutive relations for the relative permeability/saturation $k_{rw}(S_w)$ and $k_{rn}(S_n)$ as well as for the capillary pressure /saturation $p_c(S_w)$ (e.g. Helmig, 1997).

Governing equations for the one-phase/two-component flow model concept

The balance equations for the flow of two components in one fluid phase in porous media are given by the conservation of mass (equation (5)) and the Darcy Law (equation (6)). In these equations X denotes mass fraction, which is the ratio of the component mass, freshwater or saltwater, to the whole mass, and D describes hydrodynamic dispersion:

$$\frac{\partial(\phi\rho X^\kappa)}{\partial t}+\nabla\cdot\{\rho v X^\kappa-\phi D^\kappa\nabla(\rho X^\kappa)\}=\rho q \quad \text{in } \Omega xI,\kappa=f,s \tag{5}$$

$$v=-\frac{K}{\mu}(\nabla p-\rho g) \tag{6}$$

According to Oldenburg & Pruess (1995) the density of the mixture ρ can be formulated as a function of the mass fractions and the density of freshwater and saltwater (concentrated brine, see equation (7)):

$$\frac{1}{\rho} = \frac{X^f}{\rho_f} + \frac{X^s}{\rho_s} \tag{7}$$

The insertion of equation (6) into equation (5), and the consideration of equation (7), leads to two coupled nonlinear partial differential equations for the components freshwater and saltwater. For the solution, initial and boundary conditions are required.

Discretization and solvers

In this paper two-dimensional examples consisting of quadrilaterals are investigated. The partial differential equations for both model concepts are discretized in space by a finite volume based method, a so-called box scheme. For the flux terms a fully upwind technique is applied. The time is discretized with an implicit Euler scheme.

The discretizations described above lead to nonlinear and sparse systems of algebraic equations for every time step, which may have a large number of unknowns. Therefore, an outer Newton iteration is combined with an inner BiCGSTAB solver. The multigrid cycle is used for BiCGSTAB as a preconditioner. The techniques explained here as well as a number of variants are part of the modular program system MUFTE-UG for simulating multiphase flow and transport processes in heterogeneous porous media (Helmig, 1997; Helmig *et al.*, 1998).

APPLICATIONS

Single-layered system

First, the well known "Henry problem" (Henry, 1964) was chosen to simulate saltwater intrusion processes (see Fig. 1). It should be mentioned that there is an improved solution of the Henry problem determined by Segol (1994). In this example a salt tongue is entering a homogeneous, confined aquifer ($K = 10^{-9}$ m^2, $\phi = 0.35$). The top and the bottom are closed boundaries. On the left open boundary, a constant freshwater inflow ($q_f = 6.6 \times 10^{-5}$ m^2 s^{-1}, $\rho_f = 1000$ kg m^{-3}, $\mu_f = 10^{-3}$ kg m^{-1} s^{-1}) is given. On the open right boundary, a hydrostatic pressure distribution ($\rho_s = 1025$ kg m^{-3}, $\mu_s = 10^{-3}$ kg m^{-1} s^{-1}) is imposed. The domain was discretized with about 1600 elements.

In Fig. 2 the results for the two-phase flow approach are given for a linear relative permeability/saturation relationship (see e.g. Huyakorn *et al.*, 1996) and two different capillary pressure/saturation relationships. The upper part of the right boundary is open for freshwater outflow. Due to the higher density, the saltwater tongue penetrates into the system and reaches a steady state solution (see Figs 2(a), 2(b)). The freshwater flows over the saltwater and leaves the system over the upper part of the right boundary. In Fig. 2(a) the capillarity is set to zero, and consequently a sharp interface solution is obtained. Theoretically, the interface width is zero. Practically, it

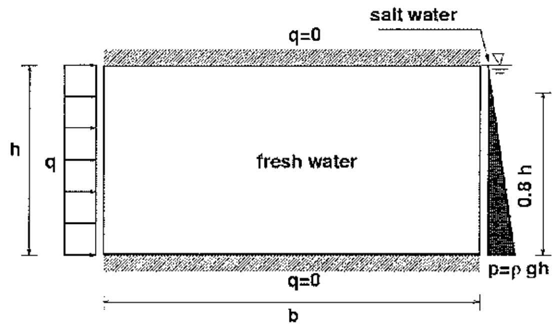

Fig. 1 System setup for the Henry problem.

is very small and determined by the element size (interpolation!). Overall, the numerical results of MUFTE-UG ($S_f = S_s = 0.5$) agree quite well with the semi-analytical results of Henry's variant without dispersion. Only the numerically computed "foot" of the sharp interface enters the system in a smaller degree. Although the capillarity is set to zero in MUFTE-UG, i.e. there is no physical dispersion, a certain numerical dispersion caused by the element size occurs. This effect decreases with decreasing element size.

In Fig. 2(b) a capillary pressure/saturation relationship of Huyakorn *et al.*, (1996), where the capillary pressure is a linear function of the density difference between fresh and saltwater, was chosen. Due to capillarity, the interface width is spreading, the isoline $S_f = S_s = 0.5$, and especially its "foot", penetrates less into the system, and a comparison with Henry's solution without dispersion makes less sense.

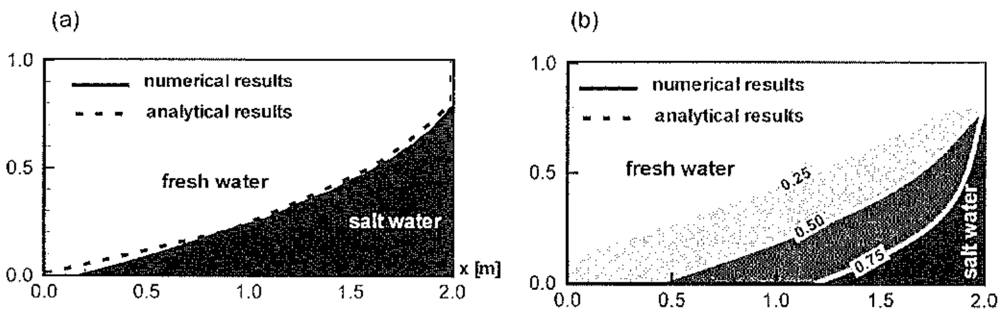

Fig. 2 (a) Two phases, $p_c(S_\alpha) = 0$. (b) Two phases, $p_c(S_\alpha) = \text{lin}$

In Figs 3(a) and 3(b) the results for the one-phase/two-component model concept are given for two different dispersivities only taking into account constant molecular diffusivities ($D = 6.6 \times 10^{-6}$, 6.6×10^{-5} m^2 s^{-1}). Figure 3(a) shows the freshwater/salt-water transition zone as well as a very good agreement of MUFTE-UG's results and Henry's variant with dispersion/diffusivity. If the diffusivity increases by one order of magnitude, the interface width increases, and the saltwater moves into the system to a smaller degree (Fig. 3(b)). If diffusivity is neglected, the results approximate the two-phase flow approach without capillarity (Fig. 2(a)). A comparison of both approaches

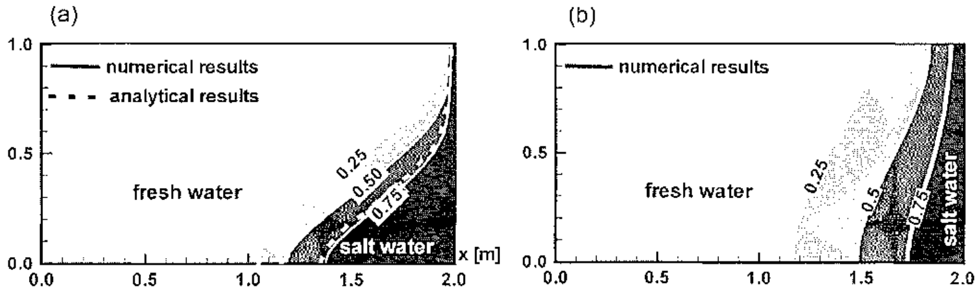

Fig. 3 (a) Two component, $D = 6.6 \times 10^{-6}$ m^2 s^{-1}. (b) Two component, $D = 6.6 \times 10^{-5}$ m^2 s^{-1}.

shows (Figs 2(a) and 3(a)) that dispersion has a significant influence on the shape and the position of the freshwater/saltwater interface.

Multilayered system

The simulation of saltwater intrusion into a multilayered system was investigated using an analytical solution of Mualem & Bear (1974) (see also Essaid, 1990, and Huyakorn *et al.*, 1996). This example (Fig. 4) is similar to the Henry problem with the difference that the aquifer system ($K = 10^{-8}$ m^2, $\phi = 0.1$) is separated by an aquitard, a thin layer ($d_2 = 0.74$ m, $t = 0.01$ m, $d_1 = 0.08$ m, $h_1 = 0.15$ m, $h_2 = 0.24$ m) with a comparatively low permeability ($K = 8 \times 10^{-11}$ m^2, $\phi = 0.1$). The top and the bottom are closed boundaries. On the right open boundary, a constant freshwater inflow ($q_f = 6.7 \times 10^{-6}$ m^2 s^{-1}, $\rho_f = 1000$ kg m^{-3}, $\mu_f = 10^{-3}$ kg m^{-1} s^{-1}) is given. On the open left boundary, a hydrostatic pressure distribution ($\rho_s = 1025$ kg m^{-3}, $\mu_s = 10^{-3}$ kg m^{-1} s^{-1}) is imposed. The domain was discretized with up to 2400 elements.

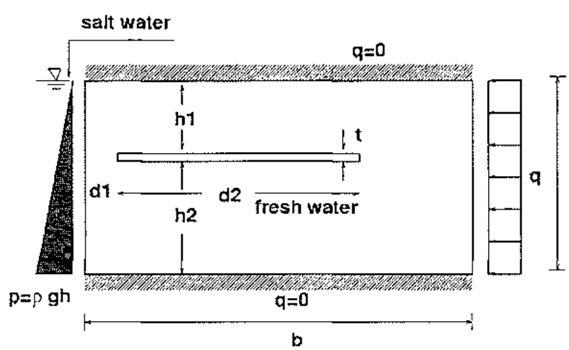

Fig. 4 System setup for the Mualem/Bear problem.

Figure 5(a) depicts the steady state solution for the two-phase flow model concept assuming a linear relative permeability/saturation relationship (Huyakorn *et al.*, 1996) and zero capillarity. Due to the higher density the saltwater flows into the system below the freshwater. The aquitard causes a jump in the run of the sharp interface in the direction of the saltwater boundary, because it nearly decouples both aquifer layers,

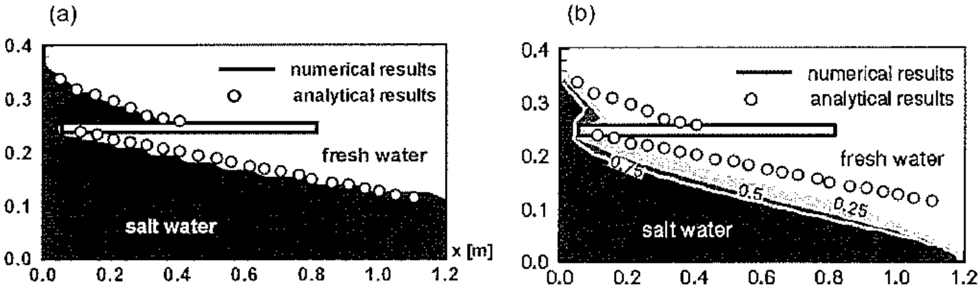

Fig. 5 (a) Two phases, $p_c(S_\alpha) = 0$. (b) Two component, $D = 6.6 \times 10^{-6}$, $t = 10$ s.

and thus the results resemble more a superposition of two Henry solutions (Fig. 2(a)). A very good agreement of MUFTE-UG's simulations and the analytical ones of Mualem & Bear (1974) was obtained.

Figure 5(b) presents the steady state solution for the one-phase/two-component approach neglecting diffusivity/dispersivity. A qualitative agreement of computed and analytical results has been obtained. The simulated interface is located below the analytically determined interface in the whole system. The computed salinity enters the aquitard, and a jump in the run of the interface as shown in Fig. 5(a) does not occur. As Mualem and Bear's solution is based on the two-phase flow model concept and the simulations on the one-phase/two-component model concept, no better agreement could be expected. Although D was set to zero, the interface width is comparatively wide. This effect is mainly caused by the density gradient in the interface region. When increasing D, the difference between the simulations and the Mualem and Bear results increase too. Overall, the results show that density driven flows are very sensitive and that the model concept must be chosen very carefully.

CONCLUSIONS

The numerical simulator MUFTE-UG is capable of dealing with saltwater intrusion processes assuming a one-phase/two-component as well as a two-phase flow model concept. Since such processes are very sensitive, the choice of the model concept has an important influence on the results. Dispersion mainly determines the shape as well as the position of the freshwater/saltwater transition zone. In the future, multiphase/ multicomponent processes, e.g. methane migration in coastal aquifer systems, will be investigated.

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