

Estimation of the sustainable yield of a borehole including boundary information, drawdown derivatives and uncertainty propagation

GERRIT VAN TONDER

Institute for Groundwater Studies, University of the Orange Free State, Bloemfontein 9301, South Africa

e-mail: gerrit@igs-nt.uovs.ac.za

HARALD KUNSTMANN

Institute for Hydromechanics and Water Resources Management, Swiss Federal Institute of Technology, ETH Hoenggerberg, Switzerland

YONGXIN XU

Department of Water Affairs, Pretoria, South Africa

FRANCOIS FOURIE

Institute for Groundwater Studies, University of the Orange Free State, Bloemfontein 9301, South Africa

Abstract A new method, the Flow Characteristic (FC) method, is presented which enables the clear characterization of flow regimes by the use of drawdown derivative information. Boundaries are accounted for with semi-analytical formulae. This approach does not require the fitting of the drawdown curve by a model. Parameter uncertainties in late time T and S values and in the distances to the boundaries are included in the analysis by applying Gaussian Error Propagation to the extrapolation of the long time drawdown curve. The FC method is verified by its successful application to a synthetically generated case study.

INTRODUCTION

An increasing number of boreholes in Southern Africa have dried up during the past years, in spite of favourable hydrological conditions. An investigation of reliable estimates for the sustainable yield of the boreholes was therefore required. Overestimation of the borehole yield was due to the application of improper extrapolation of drawdown curves, which ignored barrier boundaries and neglected parameter uncertainties arising from the imperfect knowledge of the effective aquifer properties. The following sections show how to estimate the sustainable yield of a borehole by quantifying the effects of no-flow boundaries as well as the uncertainties in the values of transmissivity, storativity and distances to the boundaries.

ESTIMATION OF THE SUSTAINABLE YIELD OF A BOREHOLE

The ratio of drawdown s to pumping rate Q is a constant for a well (if corrected for well losses). This constant only depends on the aquifer properties transmissivity T and

storativity S . If t_{long} describes the maximum operation time in which the drawdown s shall not exceed a maximum drawdown $s_{Available}$, the extrapolation of the measured pumping test drawdown can be used to determine the sustainable yield $Q_{Sustainable}$:

$$Q_{Sustainable} = Q_{Pump\ Test} \frac{s_{Available}(t = t_{long})}{s_{Pump\ Test}(t = t_{long})} \quad (1)$$

The available drawdown is for instance the position of the main water strike in the borehole. If the drawdown exceeds this position, a drastic decrease in the yield of the borehole occurs and it may dry up. The problem of extrapolating the drawdown measured during the pumping test from the time of the end of the pumping test to a time t_{long} of around two to five years, remains. This extrapolation is traditionally done by applying the Theis solution. A more sophisticated extrapolation of the pumping test drawdown beyond the time of the end of the measurement is obtained by using a Taylor series expansion based on the extrapolation of the measured drawdown curve including drawdown derivatives, and by accounting for boundaries.

Extrapolation of pumping test drawdown

The drawdown measured during a pumping test is the sum of the drawdowns due to the production well, s_{Well} , and the boundaries, $s_{Boundary}$:

$$s(t = t_{long}) = s_{Well} + s_{Boundary} \quad (2)$$

The drawdown due to the production well (s_{Well}) is extrapolated by a Taylor series expansion around the late measurement points of the drawdown at $t \approx t_{EOP}$ (subscript EOP denotes end of pumping test). The Taylor series expansion is performed with respect to the logarithm of time, \log_{10} . A second order approximation is assumed to be sufficient:

$$s_{Well}(t = t_{long}) \approx s(t = t_{EOP}) + \left. \frac{\partial s}{\partial \log t} \right|_{t=t_{EOP}} (\log t_{long} - \log t_{EOP}) + \frac{1}{2} \left. \frac{\partial^2 s}{\partial (\log t)^2} \right|_{t=t_{EOP}} (\log t_{long} - \log t_{EOP})^2 \quad (3)$$

The time t_{EOP} has to be large enough to ensure that the drawdown has already passed the early time flow behaviour that is due to well bore storage, fracture flow and double porosity effects. This can clearly be monitored by looking at the derivative plot $\partial s / \partial \log t$ (van Tonder, 1998; Bourdet et al., 1984). Usually the effect of the boundaries can only be seen at very late times of the pumping test. The extrapolation of equation (3) therefore does not in general include boundary information.

For simple geometries of the boundaries, image well theory is applied to analyse the effects of the boundaries on the drawdown ($s_{Boundary}$).

The analytical expressions and the simplified boundary configurations already yield far better estimates of the sustainable yield than the traditional Theis extrapolation, which assumes an aquifer of infinite extent. The estimate can be improved further by taking into account uncertainties in the required parameters like the late time transmissivity T , storativity S , and the distances to the boundaries a and b .

Risk analysis by uncertainty propagation

Kunstmann & Kinzelbach (1998) showed computationally efficient methods of quantifying uncertainties in groundwater modelling. The Gaussian Error Propagation method can easily and most advantageously be applied to analytical formulas. It is applied to the drawdown equations presented and described below.

The drawdown in the pumping well is a function of the parameters t, Q, T, S, a and b , where a and b are the distances to boundaries. It is assumed that the latter four parameters are not known perfectly, but are within a range around their mean values:

$$T = \hat{T} \pm \sigma_T, \quad S = \hat{S} \pm \sigma_S, \quad a = \hat{a} \pm \sigma_a, \quad b = \hat{b} \pm \sigma_b \tag{4}$$

The mean drawdown \hat{s} can be approximated by evaluating the drawdown equations at the mean values of the input parameters:

$$\hat{s} \approx s(\hat{T}, \hat{S}, \hat{a}, \hat{b}) \tag{5}$$

The standard deviation (describing the uncertainty of the drawdown) can be approximated by the following formula:

$$\sigma_s \approx \sqrt{\left(\left.\frac{\partial s}{\partial T}\right|_{T=\hat{T}}\right)^2 \sigma_T^2 + \left(\left.\frac{\partial s}{\partial S}\right|_{S=\hat{S}}\right)^2 \sigma_S^2 + \left(\left.\frac{\partial s}{\partial a}\right|_{a=\hat{a}}\right)^2 \sigma_a^2 + \left(\left.\frac{\partial s}{\partial b}\right|_{b=\hat{b}}\right)^2 \sigma_b^2} \tag{6}$$

σ_s is required at the extrapolation time t_{long} , since the uncertainty of the extrapolated drawdown is of interest. Equation (6) shows that the uncertainty σ_s is determined by the input parameter uncertainties $\sigma_T, \sigma_S, \sigma_a$ and σ_b , and the sensitivities $\partial s / \partial T, \partial s / \partial S, \partial s / \partial a$, and $\partial s / \partial b$.

The sensitivity of the drawdown with respect to the parameters is the sum of the sensitivity of the well drawdown and the sensitivity of the image wells, i.e. the boundary drawdown. In the case of transmissivity, for instance, this can be written as:

$$\left.\frac{\partial s}{\partial T}\right|_{t=t_{long}} = \left.\frac{\partial s_{Well}}{\partial T}\right|_{t=t_{long}} + \left.\frac{\partial s_{Boundary}}{\partial T}\right|_{t=t_{long}} \tag{7}$$

The well drawdown is extrapolated by a second order Taylor series expansion (equation (3)) from the end of the pumping test to the time t_{long} (that describes the maximum operation period of the borehole in the case of no recharge). Since the extrapolated well drawdown is based on a measured drawdown curve, its sensitivity with respect to the parameters cannot be calculated. The sensitivity of s_{Well} is therefore approximated by assuming a Theis sensitivity, e.g.:

$$\left.\frac{\partial s_{Well}}{\partial T}\right|_{t_{long}} \approx \left.\frac{\partial s_{Theis}}{\partial T}\right|_{t_{long}} \tag{8}$$

The analytical expression of the Theis sensitivity can easily be evaluated by a finite difference approximation. The uncertainty of the extrapolated drawdown σ_s can now be included in the estimation of the sustainable yield. The available drawdown has to be corrected by the uncertainty of the drawdown that arises from the imperfect knowledge of the aquifer parameters and the distances to the boundaries:

$$s^1_{Available} = s_{Available} - 2\sigma_s \tag{9}$$

This leads to a risk-oriented estimate of the sustainable yield.

A correction of the available drawdown by two standard deviations yields a probability of 95.5% for not exceeding the available drawdown (assuming a normal distribution for the uncertain s). A correction by one standard deviation still yields a safety of 68.3%. The owner of the borehole has to decide on the safety requirement (i.e. the probability of failure). In this manner a conservative and therefore sustainable yield should be estimated.

Application of this methodology required the determination of T and S . These parameters can be estimated by the interpretation of the drawdown curve. Moreover, to get an estimate of the available drawdown and the water strikes, the flow regime behaviour has to be investigated to identify the main fractures and the water strikes. In the following we present a new, heuristic approach for the identification of T and S and as a way to obtain better knowledge on the flow regime.

IDENTIFICATION OF CHARACTERISTIC FLOW REGIMES

Use of drawdown derivatives

A specific flow regime has a characteristic pumping test curve. The use of derivatives of pressure head has been used for many years in the oil field to evaluate flow regime characteristics (Bourdet *et al.*, 1984; Horne, 1997). Use of the derivative of pressure head *vs* time is mathematically satisfying because the derivative is directly represented in one of the diffusivity equations, which is the governing equation for all the models of transient pressure behaviour currently in use in well test analysis. Consequently, the derivative response is much more sensitive to small phenomena of interest which are all integrated and, hence, diminished, by the pressure head *vs* time solutions usually present in well test interpretations. Accurate field measurements of drawdown versus time are, however, required.

Analytical equations describing the drawdown in a borehole are of the form:

$$s = 2.3 \frac{Q}{4\pi T} \log C \quad (10)$$

where s is the drawdown in the borehole, Q is the abstraction rate of the abstraction borehole, T is the transmissivity of the aquifer system and C is a time dependent expression that varies according to aquifer type and contains the ratio T/S where S is the storativity.

From equation (10) the derivative of the drawdown with respect to $\log(t)$ is found to be:

$$\frac{\partial s}{\partial \log t} = 2.3 \frac{Q}{4\pi T} \quad (11)$$

From this equation the T value can be calculated for each time. The derivative of the logarithmic drawdown with respect to $\log(t)$ is given by:

$$\frac{\partial \log s}{\partial \log t} = \frac{1}{\ln C} \quad (12)$$

We refer to equation (11) as “derivative”, and to equation (12) as “log-derivative”. From equation (12) the ratio T/S can be calculated and the combined use of equations (11) and (12) therefore gives an S value for each time. The derivatives are calculated numerically by a linear regression line yielding the slope.

The characteristics that can be obtained from the derivative graph are as follows: Well bore storage shows a line with slope = 1 at early time. Infinite radial shows a horizontal straight-line 1 to 1.5 log cycles after well bore storage. A double porosity aquifer shows a characteristic dip after well bore storage. A single no-flow boundary shows, at most, a doubling of the derivative and two no-flow boundaries show, at most, a tripling of the derivative. A closed no-flow boundary shows a straight line with slope = 1 at late time. A recharge boundary (river or dam) shows a drastic decrease in the value of the derivative and positions of fractures are usually seen by a typical sinus wave form (i.e. at the fracture position the derivative decreases and after de-watering of the fracture the derivative increases again).

An heuristic approach for the estimation of effective T and S values

The effective T and S values are obtained by the evaluation of the derivatives as described above. We suggest taking the highest value of the drawdown derivative observed during the pumping test to estimate the effective T and S value. However, this holds only after having passed well bore storage effects and before having reached the boundaries. The reason for this heuristic approach is the following. When the water level reaches the position of a fracture, a flattening of the water level is observed. At this stage one would obtain an erroneous effective T value by using the derivative of this flattened part. The flattening of the drawdown curve is due to the fact that flow conditions have changed from confined to unconfined. At the position of the fracture, the drawdown will be according to the specific yield and not to the storage coefficient of the fracture. Because the specific yield is much higher than the storage coefficient, a flattening of the drawdown curve is observed.

The maximum drawdown derivative coincides with the maximum of the log-drawdown derivative due to the monotonic behaviour of the decimal logarithm. The effective S value curve will thus always show an upward trend in field situations. At early times, the drawdown is according to the storage coefficient of the fracture, and changes then to the specific yield at the position of the fracture. At late times the S value changes to the storativity of the matrix.

JUSTIFICATION BY SYNTHETIC EXAMPLE

The MODFLOW program was used to generate a typical pumping test solution:

Case: two-layer generated pumping test (2020×2020 m square closed boundary) with typical parameter values for fractured aquifers in the Karoo rocks of Southern Africa with a fracture zone in the bottom layer. Thickness of the first layer = 19 m and bottom layer = 1 m. Fracture zone is situated in bottom layer (220×220 m). Abstraction borehole (2 l s^{-1}) situated in the centre of this fracture zone in the bottom layer. First layer: $T_m = 1 \text{ m}^2 \text{ day}^{-1}$; $S_m = 0.001$ (typical matrix values for the Karoo

aquifers). Fracture zone $T_f = 20 \text{ m}^2 \text{ day}^{-1}$ and $S_f = 1 \times 10^{-4}$. The rest of the bottom layer has the same T_m and S_m as the top layer. Vertical $K_z = 0.1 \text{ m d}^{-1}$. Figure 1 shows the MODFLOW generated values for a period of two years.

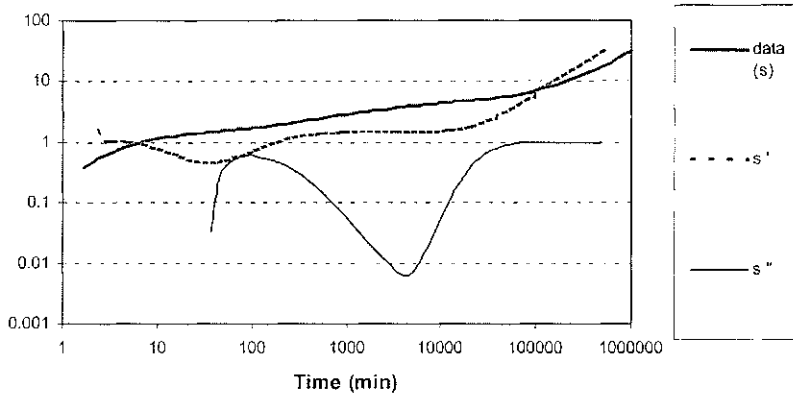


Fig. 1 MODFLOW generated data.

The use of the different derivative graphs could thus be used with great effect to identify certain specific flow characteristics of fractured-rock aquifers.

The MODFLOW program was run for a period of two years with an abstraction rate of 2 l s^{-1} . The generated data values for times up to three days (i.e. the typical length of a pumping test) were used in the FC method to estimate the drawdown and sustainable yield by extrapolating drawdown to two years. The correct answer is of course 2 l s^{-1} . Table 1 shows the results.

Table 1. Comparison between MODFLOW and FC results for synthetic generated data.

Parameter	Result: MODFLOW	FC
$s(t = t_{EOP})$ (m)	3.82	—
$s(t = t_{long})$ (m)	30.56	29.2
Q_{sust} (l s^{-1})	2	2.09

The recommended yield estimated with the FC method is within 4.5% of the MODFLOW solution. Further examples of the successful application of the method to various synthetic and real case field studies can be found in (van Tonder *et al.*, 1998).

CONCLUSIONS

The newly developed flow characteristic (FC) method allows the estimation of the sustainable yield of a borehole by incorporating boundary, drawdown derivative and uncertainty information. A comparison of the FC method results with a synthetic model indicates that the FC method can be applied with success in the prediction of

sustainable yields. A copy of the program (in the form of a Microsoft Excel spreadsheet) and documentation can be extracted from the following website: <http://www.uovs.ac.za/igs>

REFERENCES

- Bourdet, D., Ayoub, J. A. & Pirard, Y. M. (1984) Use of pressure derivatives in well test interpretation. Paper SPE 12777 presented at the 1984 SPE California regional meeting, Long Beach, April 1984.
- Horne, R. N. (1997) *Modern Well Test Analysis: A Computer-Aided Approach*. Petroway Inc., Palo Alto, USA.
- Kunstmann, H. & Kinzelbach, W. (1998) Quantifizierung von Unsicherheiten in Grundwassermodellen (Quantification of uncertainty in groundwater models). *Mathematische Geologie* **2**.
- van Tonder, G. J. (1998) *Grondwater—Een van Suid-Afrika se Belangrikste Hulpbronne van die Toekoms* (Groundwater—One of South Africa's Most Important Resources of the Future). Institute for Groundwater Studies, University of the Orange Free State, Bloemfontein, South Africa. ISBN 0-86886-597-4.
- Van Tonder, G. J., Kunstmann, H. & Xu, Y. (1998) Estimation of the sustainable yield of a borehole including boundary information, drawdown derivatives, and uncertainty propagation. *Technical Report, University of the Orange Free State, Institute for Groundwater Studies, Bloemfontein, South Africa*.