Calibration and Reliability in Groundwater Modelling (Proceedings of the ModelCARE 99 Conference held at Zürich, Switzerland, September 1999). IAHS Publ. no. 265, 2000.

Uncertainty analysis of groundwater heads around underground storage caverns due to the spatial variability of hydraulic conductivity

IL-MOON CHUNG
R & D Center, SK Engineering & Construction, 192-18 Kwanhun-dong, Jongro-du, Seoul, Korea
e-mail: imchung@cosmos.skcc.co.kr

WONCHEOL CHO & JUN-HAENG HEO
Department of Civil Engineering, Yonsei University, Seoul, Korea

Abstract Finite element methodology combined with random field theory is developed to overcome limitations of deterministic flow analysis around underground storage caverns. By using this combined model, the uncertainty of heads due to the spatial variability of hydraulic conductivity can be assessed. To determine the probability distribution for field data around underground caverns, various distributions are investigated. The Monte Carlo technique can be effectively applied to obtain an approximate solution to the two-dimensional steady flow of a stochastically defined non-uniform medium. A nearest-neighbour stochastic process model is used to generate a multilateral spatial dependence between hydraulic conductivity values in the block system. The uncertainty in model prediction depends on both the spatial heterogeneity of hydraulic conductivity and the nature of the flow system such as water curtains and boundary conditions. In particular, this uncertainty is related with the well-known gas tightness condition.

INTRODUCTION

Liquified petroleum gas (LPG) and compressed air can be stored in unlined caverns below the natural groundwater level. In order to prevent bubbles of gas from rising through the water in fractures of rock, the groundwater above the ceilings of the caverns must also flow in a downward direction and its hydraulic gradient must be greater than 1 (Åberg, 1977). This is the gas tightness condition which is essential to the maintenance of storage caverns. However, this gradient cannot be obtained by means of the natural flow of groundwater to the cavern. Instead water must be infiltrated into the rock from a system of galleries and boreholes. These boreholes are called a "water curtain". Generally, gas tightness can be determined by numerical flow analysis.

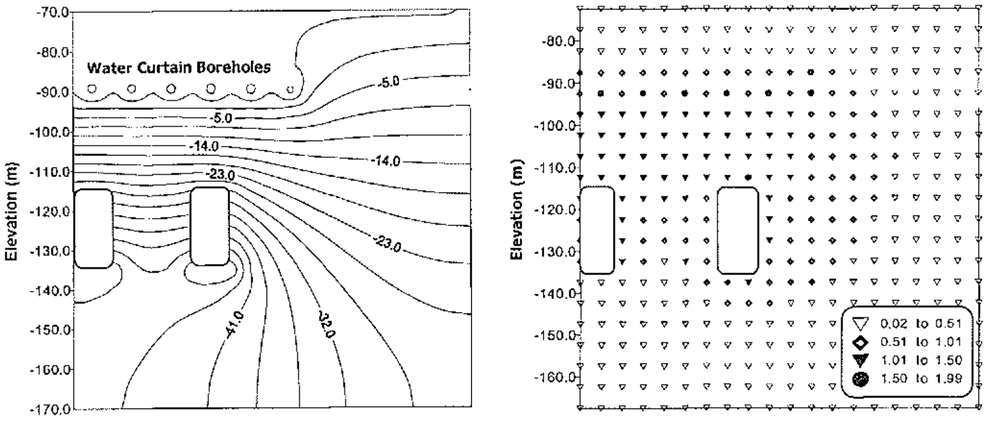
In this study, a finite element method combined with a stochastic concept is developed to analyse the uncertainty of computed heads around storage caverns due to spatial variability of hydraulic conductivity.

DETERMINISTIC MODELLING AND GAS TIGHTNESS

The governing partial differential equation is the Laplace equation in two-dimensions. The main assumption in this mathematical treatment is that the law of motion is properly described by Darcy's law in both hard rock with small fissures, and in sedimentary porous formations. In the deterministic modelling, two-dimensional flow in a vertical plane is considered. The finite element flow model used in this study was developed by Chung *et al.* (1997). The modelling area is 100 m wide and 100 m deep. The number of finite elements is 400. Linear rectangular elements are used in conjunction with a linear basis function. The mean hydraulic conductivity is 1.6×10^{-7} m s^{-1}. The water curtains are located at an elevation –90 m. The shape of the cavern is simplified as rectangular, of height 20 m and width 10 m. Two types of boundary condition (constant head boundary and no flow boundary) are used. The head value on the top boundary is 1.5 m which is the natural groundwater head. The cavern boundary also has a constant head which is the sum of the head of gas pressure (8.6 kg cm^{-2}) and elevation head. At water curtains, the constant head is maintained at 5.0 m by an artificial injection facility. Figure 1(a) shows the deterministic contours of hydraulic head. According to Åberg's criteria, gas tightness is secured by water curtains because the hydraulic gradients above caverns are greater than 1 as shown in Fig. 1(b).

PROBABILITY DISTRIBUTION OF HYDRAULIC CONDUCTIVITY IN STUDY AREA

In this study, we set up a representation of a porous medium that is a stochastic set of macroscopic elements. Within each soil type or geological unit the properties of these elements are assumed to come from frequency distributions. Thus, a proper probability density function (PDF) for the media property has to be determined. We limit ourselves to non-uniform, homogeneous isotropic soils. Various probability distributions such as two- and three-parameter gamma, GEV, Gumbel, two- and three-parameter lognormal, log-Pearson type III, are applied to determine the appropriate probability distribution model for the real field hydraulic conductivity data. There are several methods of parameter estimation such as the methods of moment, probability



(a) Head distribution. (b) Gradient distribution.

Fig. 1 Results of deterministic modelling for determination of gas tightness condition.

weighted moments and maximum likelihood. In this study, the parameters for the selected models are estimated on the basis of the method of probability weighted moments (PWM). The general form of PWM is given by (Greenwood *et al.*, 1979; Hosking, 1986):

$$M_{p,r,s} = E[X^p F^r(x)\{1 - F(x)\}^s] \tag{1}$$

where p, r, and s are integers, E represents expectation, and $F(x)$ means CDF.

To determine the appropriate probability distribution for the hydraulic conductivity data, the data of A-10 and A-12 holes in the studied area were applied. These data are listed in Table 1.

Table 1 Hydraulic conductivity data (Korea Petroleum Development Cooperation, 1985).

Hole no. A-10:		Hole no. A-12:	
Interval (m)	K ($\times 10^{-7}$ m s^{-1})	Interval (m)	K ($\times 10^{-7}$ m s^{-1})
51–54	2.595	55–58	1.830
54–57	2.664	59–62	0.498
57–60	3.139	63–66	1.036
62–65	2.020	67–70	1.235
67–70	2.651	71–74	1.356
72–75	2.273	75–78	1.333
78–81	1.894	81–84	1.201
83–86	2.273	84–87	1.263
86–89	1.894	88–91	1.544
91–94	2.146	91–94	2.092
96–99	2.273	95–98	1.449
99–102	2.020	98–101	2.183
104–107	1.998	103–106	1.731
107–110	2.188	107–110	0.429
110–113	2.399	115–118	1.115
113–116	2.399	120–123	1.709
116–119	1.832	126–129	0.857
120–123	1.768	132–135	0.943
123–136	1.427	136–139	0.772
128–131	1.768	139–142	1.544
135–138	1.768	142–145	1.614
143–146	2.378	145–148	1.499
		152–155	0.858

The PWM estimates of the parameters for each probability model are displayed in Table 2. Among the applied models, the three-parameter lognormal and log-Pearson type III models show the result of N.G., which means that estimated parameters for these two models are not within the proper range of parameters for each distribution type.

Thus, the goodness of fit tests for the five probability models are performed using the χ^2 test, Kolmogorov-Smirnov test, and Cramer von Mises test. The results of goodness of fit tests are shown in Table 3. The three-parameter gamma distribution is rejected on the basis of three goodness of fit tests, while others are accepted at the 5% significance level. In this study, the two-parameter lognormal distribution is selected as appropriate for the spatial non-uniformity of hydraulic conductivity.

Table 2 The estimated parameters for each probability distribution.

Distribution	Location parameter	Scale parameter	Shape parameter	Result
Gamma-2	0.000	0.429	0.227	O.K.
Gamma-3	–67.851	0.005	46.540	O.K.
GEV	1.535	0.635	0.355	O.K.
Gumbel	1.441	0.502	–	O.K.
Lognormal-2	0.000	0.562	0.350	O.K.
Lognormal-3	–5.952	2.042	0.080	N.G.
Log-Pearson type III	1.089	–0.306	2.000	N.G.

Table 3 The results of goodness of fit tests.

Distribution	Test	Computed value	Tabulated value	Result
Gamma-2	χ^2	0.89	7.81	O.K.
	K-S	0.09	0.18	O.K.
	CVM	0.09	0.46	O.K.
Gamma-3	χ^2	74.08	5.99	N.G.
	K-S	0.58	0.18	N.G.
	CVM	2.09	0.46	N.G.
GEV	χ^2	1.27	5.99	O.K.
	K-S	0.06	0.18	O.K.
	CVM	0.01	0.46	O.K.
Gumbel	χ^2	3.93	7.81	O.K.
	K-S	0.11	0.18	O.K.
	CVM	0.15	0.46	O.K.
Lognormal-2	χ^2	5.27	7.81	O.K.
	K-S	0.10	0.18	O.K.
	CVM	0.09	0.46	O.K.

SPATIAL VARIABILITY OF HYDRAULIC CONDUCTIVITY

To estimate the effect by spatial variability statistically, the Monte Carlo method is used (Smith & Freeze, 1979). The hydraulic conductivity values K can be chosen from the lognormal probability distribution.

The first order nearest-neighbour autoregressive relation which accounts for spatial correlation in two dimensions can be written as:

$$Y_{i,j} = \alpha_x \left(Y_{i-1,j} + Y_{i+1,j}\right) + \alpha_z \left(Y_{i,j-1} + Y_{i,j+1}\right) + \varepsilon_{i,j} \tag{2}$$

where, $Y_{i,j}$ is the random variable satisfying the nearest-neighbour relation, $\varepsilon_{i,j}$ is normal random variable uncorrelated with the other $\varepsilon_{i,j}$, α_x is an autoregressive parameter expressing the degree of spatial dependence of $Y_{i,j}$ on its two neighbouring values in the x direction, $Y_{i,j-1}$ and $Y_{i+1,j}\left(|\alpha_x| < 1\right)$, α_z is similar dependence in the z direction on $Y_{i,j-1}$ and $Y_{i,j+1}\left(|\alpha_z| < 1\right)$. The system of equations for the nearest-neighbour model can then be written as:

$$\{Y\} = \mu_y + [W]\{Y\} + \sigma_y \{\varepsilon\} \tag{3}$$

where, $[W]$ is a spatial lag operator, μ_y is the mean, and σ_y is standard deviation.

Accordingly, the lognormal generator for K_i is:

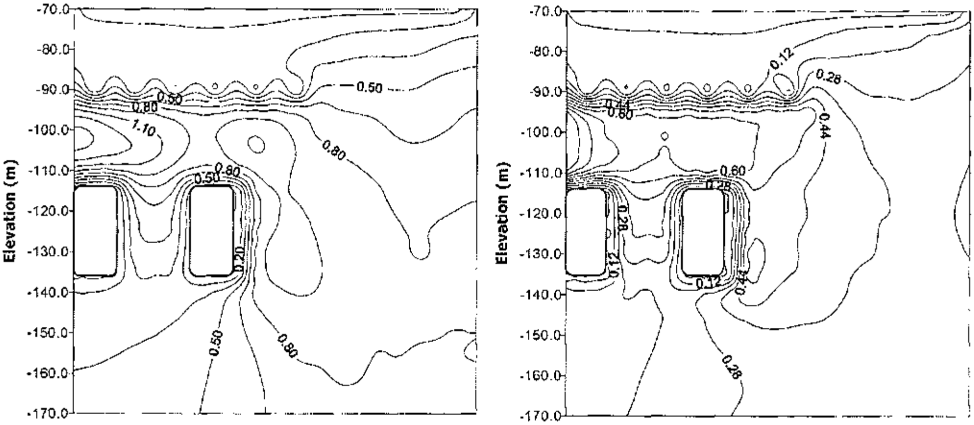
$$K_i = 10^{Y_i}, \quad i = 1, \cdots, p \tag{4}$$

where p is the number of finite elements. The FEM model is used to solve the head values on a set of nodal points within the flow domain. By repeating the analysis 1000 times, the frequency distributions of hydraulic head and gradient at the nodal points in the finite element grid can be analysed to estimate the means and standard deviations of head and gradient.

UNCERTAINTY IN HYDRAULIC HEAD

From hydraulic conductivity data in the Table 1, the log transformed mean (μ_y) and standard deviation (σ_y) can be obtained. The μ_y is –4.79 (cm s^{-1}) and σ_y is 0.19 (cm s^{-1}). To investigate the spatial dependence between neighbouring conductivity values, two examples are presented. Figure 2(a) shows the correlated case ($\alpha_x = 0.95$, $\alpha_z = 0.5$), and Fig. 2(b) shows the uncorrelated one ($\alpha_x = \alpha_z = 0$). The standard deviations of hydraulic head are approximately doubled when spatial correlation is considered.

The uncertainty in the predicted hydraulic values is greatest where the mean hydraulic gradients are relatively large, yet the region is some distance away from the known constant head boundaries, such as cavern boundaries and water curtains where the uncertainty goes to zero. In particular, the standard deviation in hydraulic head, (S_ϕ) is greatest in the upper region of the caverns.

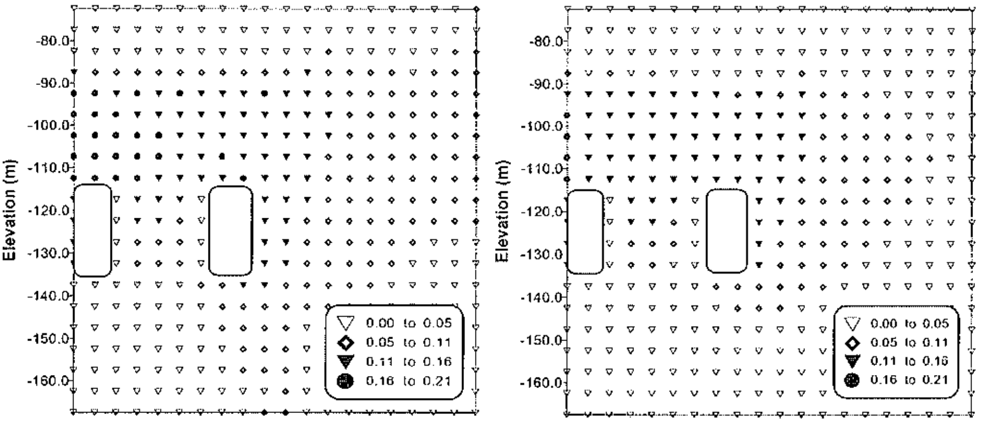


(a) Correlated case (b) Uncorrelated case

Fig. 2 The distribution of standard deviation of hydraulic head (S_ϕ).

UNCERTAINTY IN HYDRAULIC GRADIENT

A second integrated measure of the uncertainty in the model prediction is defined by the hydraulic gradient variability. The standard deviations in hydraulic gradient are plotted in Fig. 3. In the upper region of the cavern, the gas tightness can be maintained by a steady mean hydraulic gradient which is larger than 1. The results show that the gas tightness condition is directly affected by the heterogeneity of hydraulic conductivity. Therefore, if the spatial variability is very large, the gas tightness condition may not be secured. When hydraulic conductivities are correlated, the standard deviations in hydraulic gradient are larger than the uncorrelated case as shown in Fig. 3.



(a) Correlated case. (b) Uncorrelated case.

Fig. 3 The distribution of standard deviation of hydraulic head (S_g).

CONCLUSIONS

In conclusion, the two-parameter lognormal distribution is selected as an appropriate probability distribution for the real field hydraulic conductivity data. The magnitude and spatial variation of uncertainty in hydraulic head is strongly influenced by the spatial variability and the nature of the flow system within the bounded domain. It also should be noted that the gas tightness condition is affected by the spatial variability of hydraulic conductivity.

REFERENCES

Åberg, B. (1977) Prevention of gas leakage from unlined reservoir in rock. *Rockstore* 77, 399–413.

Chung, I. M., Cho, W. C. & Bae, D. H. (1997) Establishment of numerical model for groundwater flow analysis around underground caverns. *J. Korea Wat. Resour. Assoc.* **30**(1), 63–73 (in Korean).

Greenwood, J. A., Landwehr, J. M., Matalas, N. C. & Wallis, J. R. (1979) Probability weighted moments: definition and relation to parameters of several distributions expressible in inverse form. *Wat. Resour. Res.* **15**(5), 1049–1054.

Hosking, J. R. M. (1986) The theory of probability weighted moments. *Research Report RC12210, IBM T. J. Watson Research Center, Yorktown Heights, New York, USA.*

Korea Petroleum Development Cooperation (1985) The Report of Basic Investigation of "A" Storage Cavern.

Smith, L. & Freeze, R. A. (1979) Stochastic analysis of steady state groundwater flow in a bounded domain. 2. Two dimensional simulation. *Wat. Resour. Res.* **15**(6), 1543–1559.