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Influence of pumping head data in the conditional estimation of a transmissivity field

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Abstract A contribution to inverse modelling within a stochastic framework is presented. Insights are provided into the type and level of information associated with head data resulting from permanent pumping tests in a two-dimensional aquifer. A conditioning methodology based on Monte-Carlo conditional simulations is developed and applied to numerical test cases taking different data density subsets into account. Results show that satisfactory reconstruction of the original transmissivity field requires a rather high data density (regular grids of mesh size two times the correlation length of transmissivity). These results are in accordance with results formerly obtained by conditioning on head data measured within a uniform flow field. Differences between these two types of data are analysed. But a major advantage of pumping tests is that additional information is easily obtained by changing the pumping location within a given set of boreholes. This was done here and preliminary results show that important additional variance reductions are obtained.

INTRODUCTION

The problem of the incorporation of measurement data within a geostatistical inverse procedure is addressed. Usually the level of information provided in field cases does not allow for a unique identification of the transmissivity field. One is faced with a situation where the level of uncertainty associated with the determination of the transmissivity field has to be reduced by taking into account all kinds of information available for the aquifer studied. These are geological data as well as different types of measurements at some locations: transmissivity data, head data obtained within a natural flow situation or pumping tests, as well as transport data related to tracer experiments.

The focus here is on data obtained from pumping tests and the aim is to characterize the level of information provided by this type of data. The incorporation of pumping head data within an inverse procedure is addressed within the framework of geostatistical modelling for instance by Zimmerman *et al.* (1998), and received recent attention by Snodgrass & Kitanidis (1998). The point is that this type of data is generally available for most aquifers studied in addition to head data obtained within natural flow conditions. Furthermore, pumping procedure allows for multiple stimulation of the aquifer, simply by changing the location of the pumping well within a given set of boreholes (see Snodgrass & Kitanidis, 1998).

As formerly developed by Dagan (1989), for uniform flow regimes and low levels of heterogeneity, the correlation structure of transmissivity and head are computed analytically. This allows incorporation of head data into the moments of a Gaussian log transmissivity field (as well as into the velocity field or particle transport statistics) by the method of conditional probabilities. An assessment of this approach is provided in Grenier *et al.* (1996) and Grenier *et al.* (1997). For flow regimes resulting from pumping tests, the methodology cannot be applied because statistics of the flow associated with a single pumping well problem are not available. Some recent developments show the difficulties met in the computation of the stochastic PDE associated with pumping situations (e.g. Indelman & Fiori, 1997; Axness *et al.*, 1997).

We first present the methodology applied to obtain transmissivity moments conditional to pumping head data. Then we demonstrate the type and level of information provided by pumping experiments related to a unique pumping location or multiple stimulation of the aquifer due to pumping successively at different locations of the given borehole set. This is done using numerical test cases.

CONDITIONING METHOD

The inverse method is developed within the Monte-Carlo conditional simulation framework. Head data resulting from pumping tests are simply incorporated by a trial and error procedure: for a generated transmissivity field corresponding to a given Gaussian statistic the flow problem associated with pumping is simulated, then, if the heads obtained at the measurement locations agree with the heads measured from the reference field, the transmissivity field is considered an acceptable solution. Transmissivity moments are then computed by an averaging procedure on the selected fields. In this study, the generated transmissivity fields are already conditional on transmissivity measurement at the same locations so that we finally obtain fields matching transmissivity and head measurements. The rejection criterion chosen here for head data conditioning involves the difference between simulated and measured heads weighted by the inverse of the head variance function:

$$J^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{(H(x_i)-H_i)^2}{\sigma_H^2(x_i)} \qquad (1)$$

In equation (1), target function J is computed for each transmissivity field realization. N is the number of control points (head measurement locations), $H(x_i)$ stands for the calculated head at location x_i, H_i for measured head on the reference field. $\sigma_H^2(x_i)$ is the head variance computed from prior Monte-Carlo simulations. In practice, the threshold value of J for the acceptance of a transmissivity field realization is not put at zero. The choice of this limit requires finding a balance between two opposite requirements: guarantee of good selectivity by imposing a low value to the criterion J, but restraint from being too selective in order to keep a sufficient number of "conditional" realizations so as to provide statistical results. We adjusted this procedure when applying the methodology to synthetic test cases. Typically 200 selected fields are required for statistical results. Thus the method is demanding of computer time though not penalizing since 1000 realizations require roughly 10 h CPU time for a rejection rate of around 90% as reported below.

An improvement in the level of conditioning is then obtained by taking other data into account: data corresponding to other pumping stimulation for the same transmissivity field. For these additional data the conditioning procedure remains the same, the rejection criterion being extended to the larger data setup.

RESULTS

The influence of head data resulting from pumping tests is studied by applying the method to synthetic test cases involving different data grid subsets. The reference transmissivity field is a realization of a lognormal process of constant mean ($\langle Y \rangle = -4$) and unit variance. As a first step we studied the impact of head data obtained from different measurement grids for a unique pumping location in the middle of the domain (Fig. 1). In a second step, several pumping locations were introduced on a four point square geometry.

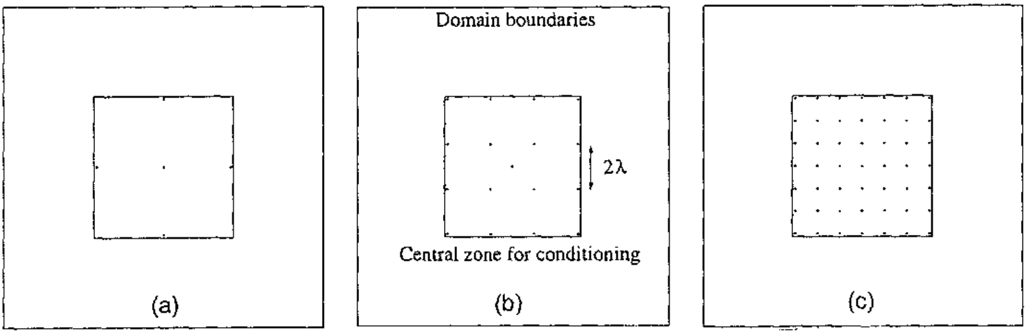

Fig. 1 Data locations for the conditioning procedure: (a) 3λ, (b) 2λ, (c) 1λ. Data are picked up from a central domain 4λ away from domain boundaries.

Illustrative results corresponding to the 2λ data grid (mesh size two times the log transmissivity correlation length λ (see Fig. 1(b)) are provided on Fig. 2. In order to avoid boundary condition effects, data grids are placed at the centre of the domain (4λ from the boundaries). Figure 2(a) provides the reference transmissivity field. Figures 2(b)–(e) provide the conditional mean transmissivity field based on transmissivity data alone (Fig. 2(d)), heads obtained in natural flow conditions (Fig. 2(e)) and from a pumping experiment at the centre of the domain. In this case, pumping at the well is simulated by means of imposed flux (Fig. 2(b)) or imposed head (Fig. 2(c)). Both types of boundary conditions led to very similar results and are not developed separately in the following. Results show that the incorporation of transmissivity as well as pumping head data on a 2λ mesh data grid allows good simulation of major patterns. Conditional variance fields (not joined) show that variance reduction is maximal in the middle of the domain.

More quantitative results are provided in Table 1: mean conditional standard deviations for different head measurement data densities as well as the corresponding rejection fraction (number of transmissivity realizations considered conditional over

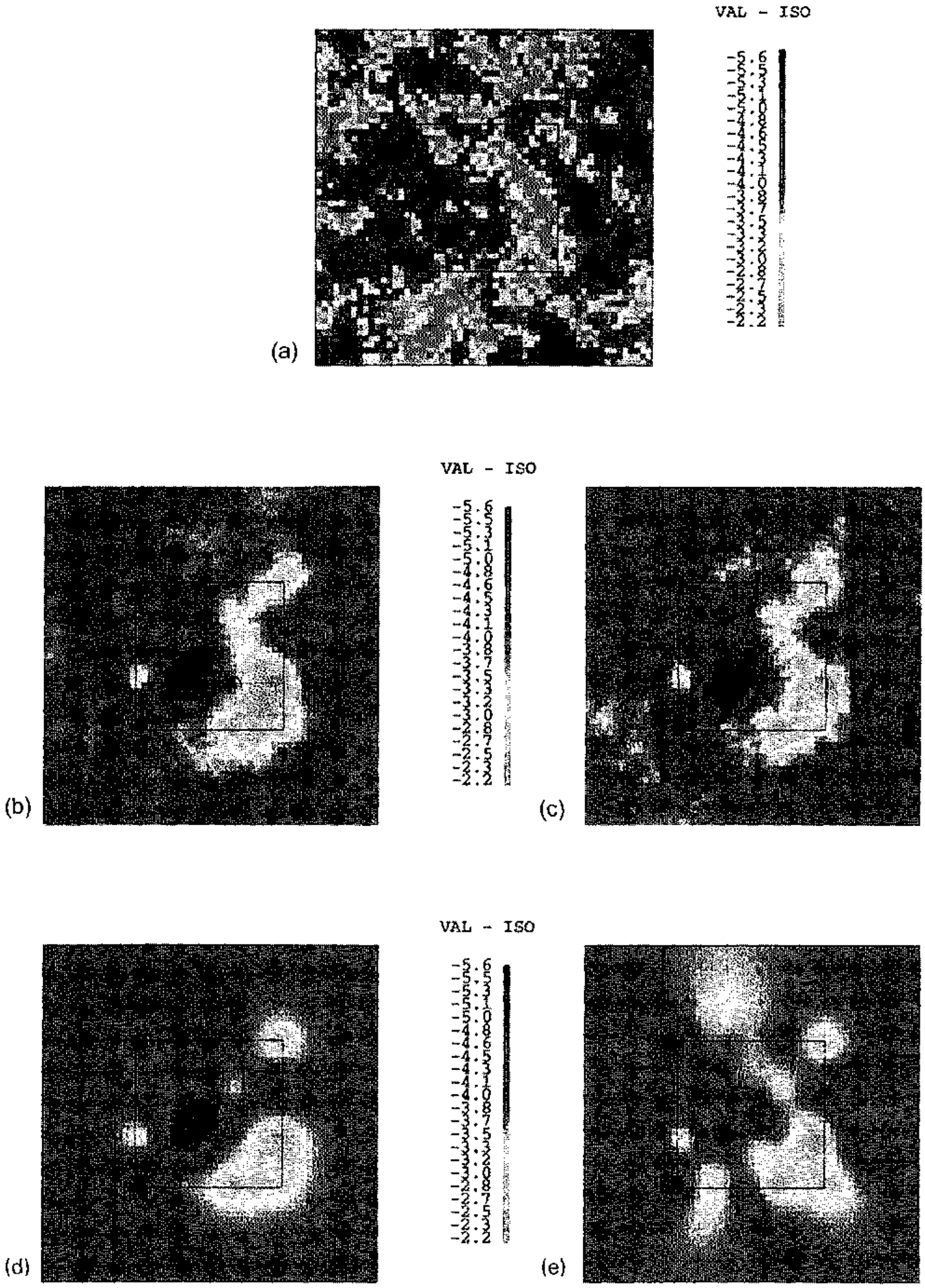

Fig. 2 Reference transmissivity field and conditional mean fields based on different types of data (2λ geometry): (a) reference (log) transmissivity field, (b) $\left\langle Y^c \middle| Y_i, H_j^{pump} \right\rangle$, measured log-transmissivity and heads resulting from pumping conditions with imposed flux at well, (c) same as (b) for imposed head at well, (d) $\left\langle Y^c \middle| Y_i \right\rangle$ measured log transmissivities, (e) $\left\langle Y^c \middle| Y_i, H_j^{unif} \right\rangle$ measured log transmissivities and heads from uniform flow conditions.

the total number of simulations). The mean is computed on the reduced domain in the centre. It first appears that the number of selected transmissivity fields is rather reduced (rejection rates over 85%). It should be noticed that the rejection rate is low for low density data since the selectivity is low. This rate turns low for high data densities because the transmissivity data taken into account in the simulation procedure have a strong conditioning effect. The level of variance reduction obtained is roughly comparable though a little bit smaller than what is obtained by incorporating uniform flow head data measurements at the same locations. A data density corresponding to the 2λ configuration is required to achieve a significant variance reduction. Head data obtained in both types of flow provide long range variance reductions while transmissivity data have a strong but local variance reduction effect. This explains why the introduction of head data does not lead to a major variance reduction for high density data grids: 1λ and 2λ (see Grenier *et al.*, 1996). The zones of influence for each type of head data are differently distributed: in the case of a pumping test, the largest reduction is achieved between the pumping well and the head observation location. Grenier *et al.* (1996) provide results related to natural flow head data.

Table 1 Influence of data type and density on mean conditional standard deviation (mean cond. std. dev.). Associated rejection rates.

	3λ	2λ	1λ
Heads from pumping test: mean cond. std. dev.	0.74	0.62	0.39
Simulation rejection rate	93%	94%	87%
Heads from natural flow: mean cond. std. dev.	0.68	0.51	0.28
Transmissivity data: mean cond. std. dev.	0.83	0.67	0.40

This field characterization strategy is improved by keeping the same well locations while pumping successively from different wells. This provides additional head measurements and leads to a more efficient selection of transmissivity fields within the inverse procedure. Results provided in Table 2 show that such a cross test pumping strategy promisingly increases the level of variance reduction. These results correspond to a four point square geometry. Pumping experiments involve successively: one location, two diagonal locations, all four locations. The square size is 4λ here, a configuration for which the impact of transmissivity measurements is very low.

Table 2 Influence of additional information resulting from different pumping locations in terms of mean conditional standard deviation (mean cond. std. dev.). Associated rejection rates.

4λ geometry	One pumping location	Two pumping locations	Four pumping locations	Natural head gradient	Transmissivity measurements only
Mean cond. std. dev.	0.8	0.75	0.65	0.77	0.86
Rejection rate	85%	97%	99%	–	–

It appears that data originating from different pumping locations lead to a higher selectivity among the transmissivity fields. Variance reduction for two pumping locations with mesh size 4λ is roughly similar to that observed for the former central pumping location and a 3λ mesh size grid. For all four pumping locations, the variance reduction is better than for the previous 3λ grid data set.

CONCLUSIONS

We have developed a methodology providing conditional moments of transmissivity taking steady state pumping head data into account. This method proved operational on numerical transmissivity fields.

Results show that head data provide long scale information on transmissivity fields. This is particularly interesting for cases of scarce data. For a regular grid of measurements of a grid size roughly two times the log transmissivity correlation length, a significant variance reduction can be achieved and major features of the transmissivity field are captured. Furthermore, an increased variance reduction is obtained by adding head data measured from pumping tests made at various well locations. For instance, for the 4λ grid, the mean conditional standard deviation obtained taking all pumping locations into account is roughly comparable to the same quantity computed for one single pumping location and denser head measurement grids (2λ to 3λ mesh size).

This type of multiple conditioning appears promising. However, the conditional method used here could become expensive in terms of computation costs. According to ongoing work, incorporation of tracer data from pumping tests, such as tracer arrival time, appears to increase significantly the quality of the estimation.

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