

Permeability field estimation by conditional simulations of geophysical data

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Abstract A new method for the determination of permeability fields in highly heterogeneous aquifers is proposed. It stems from the known relations between soil electric resistivity and permeability. Several cut-offs are made to electric resistivity probability distribution. These cut-offs are heuristic estimates about what classes of values of one variable should relate to what classes of values of the other. The outcome of this process is a set of cumulative indicator variables. These variables are then simulated. The permeability fields are obtained by the intersection of the indicator variables. These permeability fields may be used in the development of conceptual models; autocorrelation distances (integer scales) obtained on the permeability fields may be used as input to macrodispersivity models. The method was applied to the karst aquifer of the Escarpão, in central Algarve, Portugal. The images obtained fitted well the geological structures identified in the fieldwork. The results of this method may be useful for the selection of new drilling spots, as input to flow and solute transport models, for the *a priori* determination of macrodispersivity parameters, and of the fractal behaviour of permeability distribution. This approach may be a very useful tool for aquifer parameterization when the available information is scarce.

INTRODUCTION

Several studies incorporate indirect information to estimate hydraulic parameters of aquifers. For example: Ahmed *et al.* (1988) and Ahmed & Marsily (1987) estimated transmissivity by co-kriging with transversal resistivity; Copty *et al.* (1993) used a Bayesian method to estimate permeabilities (k) through seismic wave propagation velocities; Schafmeister & Burger (1995) used indicators to estimate hydraulic conductivities (K); Schafmeister (1996) applied the indicator coding to estimate K from lithological data; Garcia & Froidevaux (1996) co-kringed the indicators to generate contamination probability maps; Dimitrakopoulos & Dagbert (1993) and Muge *et al.* (1997) used sequential simulation of the indicator to generate permeability fields in a manner similar to the one proposed here, but only with direct data (lithological). We propose a method for obtaining estimates of the permeability fields in highly heterogeneous media (fractured/karstified), to be used when real data is scarce or non-available. The permeability (k) fields are estimated by conditional simulation of the indicator, which is dependent only on geophysical data where knowledge of the apparent electric resistivity (ρ_a) is compulsory. The method is general for geophysical methods as long as they provide information about the number

and thickness of the lithological layers and their electric resistivity. It was tested for Very Low Frequency–Resistivity. This method may be useful when the spatial distribution of the permeability is not known, nor its heterogeneity and isotropy. Karst systems are therefore good candidates. In the form presented here it should only be applied in cases where direct information is not available, nor other indirect data. If K or T values should be present, then co-simulation should be used, which requires a very small change in the method described here. The method was applied to a karst aquifer in the south of Portugal (Algarve): the Escarpão aquifer, in the area surrounding the Albufeira landfill.

Relationships between electrical resistivity and aquifer permeability

Despite the fact that different physical laws describe electrical current and groundwater flow, some analogies have been established between them. Some authors have determined empirical relations, linear or logarithmic, between hydraulic conductivity and apparent electrical resistivity ($K = f(\rho_a)$), or between transmissivity and transverse resistance, R , ($T = f(R)$). $R = \rho H$, where ρ is the formation electric resistivity, H is the aquifer thickness (or lithological layer thickness). The longitudinal conductance, C_e , is defined as $C_e = \rho/H$, or $C_e = \sigma H$, with σ the electrical conductivity. These relationships are based on the relation between Darcy's law and Ohm's law (equation (1))—see Table 1—where Q is the yield and J the hydraulic gradient; A is the cross sectional area perpendicular to the flow; i is electrical current density; σ is the electrical conductivity $\equiv 1/\rho$, and E is the potential. Ohm's law relates the electrical current, potential difference and the electrical resistivity, so that $\partial V = \partial RI$, and $\partial R = (\rho \partial L / \partial A)$; replacing gives $\partial V / \partial L = -\rho I / \partial A$, where the left term represents the gradient through the element of length ∂L , in $V m^{-1}$; I is in $A m^{-2}$. Considering a prism of aquifer material with unit cross sectional area, and thickness H , the two laws can be combined (Sri Nivas & Singhal, 1981; equation (2)). Maillet (1947) named R and C_e as the Dar-Zarrouk variable and function, respectively, which together form the Dar-Zarrouk parameters. Other authors used an adimensional parameter obtained by the division of electrical resistivity, ρ , by the water electrical resistivity, ρ_w , giving rise to the formation factor, F , (Archie (1942; equation (3)). That is, there is a relationship between F and porosity, n , with m and a constants related to the medium. Archie (1950) established the relationship between permeability, k , and porosity expressed by equation (4).

Porosity may be calculated by the second equation in (3) and replaced in (4) to obtain a permeability estimate from the electrical resistivity of the medium and the fluid. This relation is fundamental for the development of many other models

Table 1 Equations relating geophysical and hydrological parameters. See text for nomenclature.

Equation	(1)	(2)	(3)	(4)
	$Q = K J A$	$T = K \sigma R$	$F = \frac{\rho}{\rho_w}$	$k = b n^c$
	$i = \sigma E$	$T = \frac{K}{\sigma} C_e$	$F = a n^{-m}$	b and c are constants

based on the correlation between n and ρ (e.g. Urish, 1981; Kosinski & Kelly, 1981); between ρ and K (e.g. Heigold *et al.*, 1979); between R and T (Sri Nivas & Singhal, 1981, 1985); and between F and k (Kwader, 1985). The Very Low Frequency–Resistivity (VLF–R) is mentioned because it was used in the case study described later in the text. The basis of the method can be found in most geophysical textbooks. Instrument sensitivity and precision limited for some time the application of VLF–R methods to karst media. However, Müller (1982, 1983) showed encouraging results with devices developed specifically for groundwater research. Further developments enabled operating in the 12 to 240 kHz range, more suited for the required purposes. In fissured, fractured and/or karstified formations, the distinction between altered (potentially more aquifer) and non-altered rock is usually possible with this method.

Stochastic simulations

In the Sequential Indicator Simulation (SIS) the original values are transformed into indicators which are defined for established cut-off values (based on the distribution function or any other relevant information). Conditional distributions are computed for each estimated node by simple kriging of the indicators. If indicator variables are not independent then disjunctive kriging should be used. Sequential methods have several advantages, especially when dealing with anisotropies and conditioning.

ESTIMATION OF THE POTENTIAL PERMEABILITY FIELDS

The methodology here proposed should be applied only when no other data but geophysical data (phase and electrical resistivity) is available. The main objective is to estimate potential permeability fields in highly heterogeneous aquifers. The definition of potential permeability will be discussed later. These fields may be used as input state variables into simulation models, or used to estimate dispersive parameters of aquifers (Nunes, 1998). The interpretation of apparent electrical resistivity (ρ_a) and phase (ϕ) pairs for different wave frequencies allows the estimation of the lithostratigraphic sequence and the real electrical resistivity (ρ) of each layer—inversion procedure. The method proposed considers models that estimate both real electrical resistivity ($\rho(\mathbf{x})$) and layer thickness ($h(\mathbf{x})$). The number of layers is a function of the phase inversions detected between different wave frequencies, and naturally of the skin depth (exploration depth). After the inversion of the geophysical data ($\rho_a(\mathbf{x})$, $\phi(\mathbf{x})$, $\mathbf{x} = (x, y)$), a matrix Δ' ($\Delta'_{N \times 2} = \{\rho(x_i, x_j, l), h(x_i, x_j, l)\}$, $x_i, x_j =$ coordinates of sampled or estimated data points, $l = 1, \dots$, number of lithofacies at the point) of real electrical resistivities, ρ , and layer thickness, h . If layer thickness is divided in segments of equal dimension, a three-dimensional data set is obtained. So, the VLF–R data set, in the form of apparent electrical resistivity and phase pairs for each wave frequency, is inverted with an inversion model (see e.g. Fischer & Le Quang (1981), or Kaufman & Keller (1981)), and new information about the number and thickness of the layers detected between the surface and a limiting depth—the skin depth—and their real electrical resistivity is obtained for all the initial data points. By dividing the layers' thickness into segments and assigning to their centre the real

electrical resistivity value of the layer, then a depth property variation is added to the horizontal information. It is proposed that heuristic relations between ρ and ϕ be used. These relations are based on the modeller's experience and the physical phenomena under study. Direct relations are made between intervals of electrical resistivity and intervals of potential permeability—that is relations between classes. N cut-off values are defined on the variable ρ , $\rho_1, \rho_2, \dots, \rho_N$, resulting in N indicator variables $I_i(x)$:

$$I(x, \rho_i) = \begin{cases} 1 & \text{if } \rho(x) \leq \rho_i \\ 0 & \text{if } \rho(x) > \rho_i \end{cases} \quad \text{with } i = 1, \dots, N \text{ cut-off values} \quad (5)$$

One advantage of using cumulative indicator variables is that the indicator co-kriging may no longer be necessary; another is illustrated by the method described above for determination of transition probabilities. To determine the transition probabilities experimental variograms for all the important directions in each indicator variable are determined, as well as the ratios between cross-variograms and simple variograms (Rivoirard, 1990, 1994). Indicator fields are estimated (by kriging or simulation) with the method that best fits the properties identified in the identification procedure. The outcome is N separate cumulative indicator fields. But we are interested in the potential permeability on each estimated point. With the N cut-off values, $N + 1$ non-cumulative variables, $I'(x)$, may be drawn: lower than the lowest cut-off, in between the former and the second lowest cut-off, and so on until the last cut-off; and one variable with values higher than the highest cut-off. Each of the former variables is associated with a well defined electrical resistivity interval, and therefore represents a well defined potential permeability class (where $k_p^i(x)$ are potential permeabilities):

$$I'_i(x) = k_p^i(x), \quad i = 1, \dots, N + 1 \quad (6)$$

APPLICATION TO A CASE STUDY

The proposed methodology was applied to the karst Escarpão aquifer in the south of Portugal, the Algarve region. The Escarpão formations are part of a sequence of limestones and dolomites of the Kimmeridgian Middle-Portlandian, with an average thickness of 700 m. They are highly fractured and karstified, especially in the more dolomitized areas, and therefore have good hydraulic properties. The upper part of the sequence is less permeable. Groundwater flow occurs essentially through the fractures and dissolution channels, with preferential development in the N20 and N140 directions (Almeida, 1985). Under this formation are the marls of the Peral that form the basement of the aquifer.

The VLF-R profiles were made at 19, 77.5 and 183 kHz with a sources direction of N30, with a device with a frequency range extended up to 300 kHz. Profiles were made in the east-west direction, therefore favouring the detection of structures with a north-south alignment, but the vertical soundings may mask their influence. The spacing between sounding points was kept as close as possible to 25 m. Electrode spacing was constant and equal to 5 m (sampling precision). The field data collected were out of phase (phase), ϕ , and apparent electrical resistivity, ρ_a , at each of the frequencies. The following heuristic relations between ρ and k_p were assumed:

- $\rho < 55 \Omega \text{ m} \Rightarrow$ highly conductive fractures or karst channels (very high potential permeability);
- $55 < \rho < 110 \Omega \text{ m} \Rightarrow$ less conductive fractures and karst conduits, smaller dimensions (high permeability);
- $110 < \rho < 220 \Omega \text{ m} \Rightarrow$ small fractures in the blocks (low permeability);
- $\rho > 220 \Omega \text{ m} \Rightarrow$ blocks, unaltered rock (very low permeability).

The classification was therefore: k_p^1 : Very high permeability; k_p^2 : High permeability; k_p^3 : Low permeability; k_p^4 : Very low permeability. These classes of permeabilities are represented in Fig. 1 as white, light grey, dark grey and black. The areas of higher permeability are in lighter colours, allowing a fast visual interpretation of the potential location of groundwater carrying conduits.

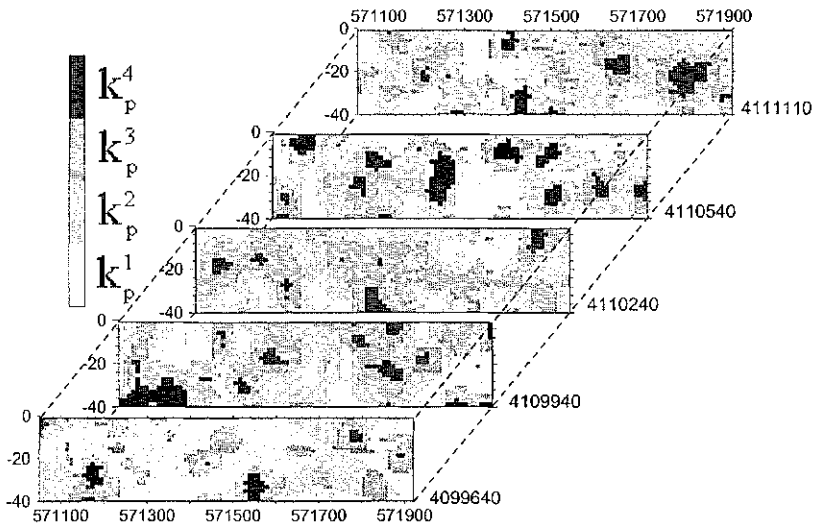


Fig. 1 Five east-west vertical cuts along the north-south axes. All values in metres (UTM co-ordinates). k_p : potential permeabilities (see text).

Results

The results are in good agreement with the known geology of the area and are therefore very encouraging for further developments and testing. Figure 1 shows five vertical east-west cuts along the north-south direction, between the east-west coordinates UTM 571000 and 571900, and north-south UTM 4099640 and 4111110. The depth is constant and equal to 40 m below surface. The images were obtained from a three-dimensional field. The central area of Fig. 1 represents a more conductive north-south channel expected from the known geology of the area. This channel is in fact composed of several areas of higher permeability intermingled in lower conductivity materials as a result of strong fracturing and karstification.

These results are very encouraging for the practical application of such a method to obtain permeability fields from geophysical data. Future developments must include the crossed analysis of other variables (in the framework of disjunctive kriging). The other variables may include, e.g. geophysical data, K or T , h , and water quality

parameters. Stochastic simulations may generate several three-dimensional potential permeability fields, used as input for the definition of conceptual models. It is therefore easier to estimate water movement and contaminant transport in areas where the available hydrogeological information is not available. However, the estimates will be prone to small scale errors (high variability resulting from large small-scale variance, or in geostatistical terms, due to large nugget effects)—in the sense that only medium to large-scale properties are expected to be reproduced. A disadvantage of the VLF-R method is its shallow skin depth in the presence of very conductive overburdens. The data shown above indicates a penetration of 40 m, which in many cases may not be enough to reach the saturated zone. Higher penetration depth is possible if portable VLF sources are used. Unfortunately the amount of problems introduced with these sources is so high that their use is not possible in many cases (e.g. where horizontal electromagnetic wave propagation can not be assumed). Higher skin depths are attainable for more resistive media. The average electrical resistivity of the limestone and dolomites in the study area is 200 Ω m, which is at the lower end of the scale for the earth materials. The method is therefore of general application, but limited for particular cases.

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REFERENCES

- Ahmed, S. & Marsily, G. (1987) Comparison of geostatistical methods for estimating transmissivity using data on transmissivity and specific capacity. *Wat. Resour. Res.* **23**(9), 1717–1737.
- Ahmed, S., Marsily, G. & Talbot, A. (1988) Combined use of hydraulic and electrical properties of an aquifer in a geostatistical estimation of transmissivity. *Ground Water* **26**(1), 78–85.
- Almeida, C. (1985) Hidrogeologia do algarve central. PhD Thesis, Dep. de Geologia, Faculdade de Ciências de Lisboa, Lisboa, Portugal.
- Archie, G. E. (1942) The electrical resistivity log as an aid in determining some reservoir characteristics. *Am. Inst. Min. Metal. Petrol. Engng Tech. Report 1422*.
- Archie, G. E. (1950) Introduction to petrophysics of reservoir rocks. *Bull. Am. Assoc. Petrol. Geologists* **34**, 943–961.
- Coply, N., Rubin, Y. & Mavco, G. (1993) Geophysical-hydrological identification of field permeabilities through Bayesian updating. *Wat. Resour. Res.* **29**(8), 2813–2825.
- Dimitrakopoulos, R. & Dagbert, M. (1993) Sequential modelling of relative indicator variables dealing with multiple lithology types. In: *Proc. Geostatistics Troia '92* (ed. by A. Soares), 413–424. Kluwer Academic Publishers, Dordrecht, The Netherlands.
- Fischer, G. & Le Quang, B. V. (1981) Topography and minimization of the standard deviation in one-dimensional magnetotelluric modelling. *Geophys. J. R. Astron. Soc.* **67**, 279–292.
- Garcia, M. & Froidevaux, R. (1996) Application of geostatistics to 3D modelling of contaminated sites: a case study. In: *Pre-prints GeoENV96* (First European Conference on Geostatistics for Environmental Applications, Lisbon, November, 1996).
- Graham, W. & McLaughlin, D. (1989) Stochastic analysis of nonstationary subsurface solute transport. 2 Conditional moments. *Wat. Resour. Res.* **25**(11), 2331–2355.
- Heigold, P. C., Gilkeson, R. H., Cartwright, K. & Rud, P. C. (1979) Aquifer transmissivity from surficial electrical methods. *Ground Water* **17**, 338–245.
- Kaufman, A. A. & Keller, G. V. (1981) *The Magnetotelluric Sounding Method*. Elsevier, The Netherlands.
- Kosinski, W. K. & Kelly, W. E. (1981) Geoelectric soundings for predicting aquifer properties. *Ground Water* **19**(2), 163–171.
- Kwader, T. (1985) Estimating aquifer permeability from formation resistivity factors. *Ground Water* **23**(6), 762–766.
- Maillet, R. (1947). The fundamental equations of electrical prospecting, *Geophysics* **12**, 529–556.

- Muge, F., Ribeiro, L., Pina, P. & Oliveira, V. (1997) Morphological characterization of the spatial variability of hydrostratigraphic units. In: *Geostatistics Wollongong '96*, vol. 2 (ed. by E. Y. Baafi & N. A. Schofield), 1134–1145. Kluwer Academic Publishers, Dordrecht, The Netherlands.
- Müller, I. (1982) Rôle de la prospection électromagnétique VLF pour la mise en valeur et la protection des aquifères calcaires. *Annales Scientif. de l'Université de Besançon, 3em Coll. d'Hydro. en Pays Calcaire, Besançon 1*, 219–226.
- Müller, I. (1983) Anisotropic properties of rocks detected with electromagnetic VLF. In: *Proceedings International Symposium on Field Measurements in Geomechanics* (Zurich, September 1983), 273–282. Zurich, Switzerland.
- Nunes, L. M. (1998) Estimativa do campo de permeabilidades potenciais e dos tensores de macrodispersividade em aquíferos heterogêneos a partir de informação geofísica. MSc Thesis, IST-UTL, Lisboa, Portugal.
- Rivoirard, J. (1990) *Introduction to Disjunctive Kriging and Nonlinear Geostatistics*. Centre de Geostatistique, Ecole des Mines de Paris, France.
- Rivoirard, J. (1994) *Introduction to Disjunctive Kriging and Non-linear Geostatistics*. Clarendon Press, Oxford, UK.
- Scheifmeister, M. Th. & Burger, H. (1995) Merging quantitative and qualitative information as input to contaminated flow simulation: a case study. In: *International Meeting of IAMG* (Osaka, October–November 1995), Abstract Volume for The Technical Programs, 43–45.
- Scheifmeister, M. Th. (1996) Parameter estimation for groundwater models by indicator kriging. In: *Pre-Print GeoENV96* (First European Conference on Geostatistics for Environmental Applications, Lisbon, November 1996).
- Sri Nivas & Singhal, D. C. (1981) Estimation of aquifer transmissivity from dar-zarrouk parameters in porous media. *J. Hydrol.* **50**, 393–399.
- Sri Nivas & Singhal, D. C. (1985) Aquifer transmissivity of porous media from resistivity data. *J. Hydrol.* **82**, 143–153.
- Urish, D. W. (1981) Electrical resistivity-hydraulic conductivity relationships in glacial outwash aquifers. *Wat. Resour. Res.* **17**(3), 1401–1408.