

The use of well transport theory for identification of aquifer properties: 1. Radial divergent flow, 2. Recharging and pumping wells flow

G. DAGAN

*Department of Fluid Mechanics and Heat Transfer, Faculty of Engineering,
Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel*
e-mail: dagan@eng.tau.ac.il

P. INDELMAN

Faculty of Civil Engineering, Technion, Technion City, Haifa 32000, Israel

Abstract Transport of conservative and reactive solute by flow from a recharging well, and by flow between a recharging and a pumping well, is analysed. The macrodispersivity for transport in the radial flow is shown to be smaller by a factor of three than in the uniform flow. The breakthrough curves in the pumping well are determined as functions of time and the heterogeneity of the formation.

MATHEMATICAL STATEMENT OF THE FLOW AND TRANSPORT PROBLEMS

We consider transport of a plume of an inert solute by water flowing through natural, large scale, heterogeneous formations. The conductivity $K(\mathbf{x})$ is regarded as a random space function. In the past, considerable effort has been invested in solving transport under conditions of uniform mean flow (e.g. Dagan, 1989) that pertain to natural gradient flows. However, there are important applications in which the mean flow varies rapidly in space. We refer to wells that inject or pump water carrying a solute. Such problems arise for instance during injection of contaminants, in defining the protection zones of pumping wells, in remediation schemes. Non-uniform flows in heterogeneous media have been studied to a much lesser extent than uniform flows. A few statistical moments of flow variables were investigated by Indelman *et al.* (1996) and their results serve as background for the present study. In particular we adopt the same simplifying assumptions regarding the well model.

The problem here is to determine the spreading pattern of the solute, as affected by the random variation of $K(\mathbf{x})$. In spite of its importance, there are no theoretical investigations of this topic in the literature, which deals exclusively with the effect of pore-scale dispersion pertaining to homogeneous media (e.g. Chen, 1987). The objective of the present study is to investigate advective transport for two typical configurations: steady and diverging radial flow, pertaining to flow from injecting wells and steady flow between a recharging and a pumping well.

The equations of flow are Darcy's law and the continuity equation $\mathbf{q} = -K\nabla H$ and $\nabla \cdot \mathbf{q} = 0$, respectively, where \mathbf{q} and H are the specific discharge and the pressure head, respectively, while $Y = \ln K$. Let $\langle Y \rangle = \ln K_G$, and $C_Y(\mathbf{x}', \mathbf{x}'') = \sigma_Y^2 \rho_Y(\mathbf{x}' - \mathbf{x}'')$, be the mean and the correlation function of the log conductivity, Y , which is assumed given.

We assume axisymmetric heterogeneity determined by the horizontal I and vertical L_v integral scales. Once the flow equations are solved, the velocity field is given by $\mathbf{V}(\mathbf{x}) = -\mathbf{q}/n = K\mathbf{E}/n$ where $n = \text{const}$ is the porosity and $\mathbf{E} = -\nabla H$.

The concentration satisfies the transport equation:

$$\frac{\partial C}{\partial t} + \mathbf{V} \cdot \nabla C = \nabla \cdot (\mathbf{D}_d \nabla C) \tag{1}$$

We assume instantaneous injection, i.e. $C(\mathbf{x},0) = C_0$ within a volume V_0 and $C(\mathbf{x},0) = 0$ outside of V_0 . In equation (1) \mathbf{D}_d is the pore-scale dispersion tensor. Here we consider media that are isotropic at the pore scale, so that \mathbf{D}_d is determined by transverse D_{dT} and α_{dT} and longitudinal D_{dL} and α_{dL} dispersion coefficients and dispersivities, respectively. We follow the Lagrangian approach in solving the transport problem and express the solution of equation (1) with the aid of the trajectories $\mathbf{x} = \mathbf{X}_t(t; \mathbf{a}) = \mathbf{X}(t; \mathbf{a}) + \mathbf{X}_d(t)$, where \mathbf{X}_d is associated with a Brownian motion type of transport. The solution of equation (1) can be written conveniently as follows:

$$C(\mathbf{x},t) = \int_{V_0} C_0(\mathbf{a}) \delta[\mathbf{x} - \mathbf{X}_t(t; \mathbf{a})] d\mathbf{a} \tag{2}$$

Here we limit the study to large V_0 at the heterogeneity scale, permitting us to adopt the ergodic hypothesis. Then, the first and second concentration moments of the plume are given by $\mathbf{R} \equiv \langle \mathbf{R} \rangle = n/M \int_{V_0} \langle C_0(\mathbf{a}) \mathbf{X}(t; \mathbf{a}) \rangle d\mathbf{a}$ and

$$S_{ij} \equiv \langle S_{ij} \rangle = n/M \int_{V_0} \langle C_0(\mathbf{a}) (X_i - R_i)(X_j - R_j) \rangle d\mathbf{a} \quad (\text{see e.g. Dagan, 1989}).$$

The derivatives $\mathbf{U} = d\mathbf{R}/dt$ and $D_{ij} = (1/2)dS_{ij}/dt$ define the plume mean velocity and apparent "macrodispersion" coefficients, respectively, whereas in conditions of mean uniform flow $\alpha_{ij} = D_{ij}$ are "macrodispersivities".

Both flow and transport problems are difficult to solve exactly. Approximate, simple results were obtained in the past by adopting a first-order approximation in the log conductivity variance σ_Y^2 . Thus, the head gradient is expanded as $\mathbf{E} = \mathbf{E}^{(0)} + \mathbf{E}^{(1)} + O(\sigma_Y^2)$, leading to the mean velocity $\mathbf{U} = K_G/n\mathbf{E}^{(0)} + O(\sigma_Y^2)$ and the velocity fluctuation $\mathbf{u} = K_G/n(Y'\mathbf{E}^{(0)} + \mathbf{E}^{(1)}) + O(\sigma_Y^2)$. The two-point velocity covariances $u_{ij}(\mathbf{x}, \mathbf{x}') = \langle u_i(\mathbf{x})u_j(\mathbf{x}') \rangle$ are now simply derived in terms of \mathbf{E} and Y' . Finally, after expanding $\mathbf{X}_t = \langle \mathbf{X} \rangle + \mathbf{X}' + \mathbf{X}_d$ in the argument of $\mathbf{V} = \mathbf{U} + \mathbf{u}$, the trajectories satisfy in leading order the following equations:

$$\frac{d\langle X_i \rangle}{dt} = \frac{d\langle \mathbf{X} \rangle}{dt} = \mathbf{U}(\langle \mathbf{X} \rangle); \quad \frac{d\mathbf{X}'}{dt} = \mathbf{u}(\langle \mathbf{X} \rangle + \mathbf{X}_d) + X'_i \frac{\partial \mathbf{U}(\langle \mathbf{X} \rangle + \mathbf{X}_d)}{\partial \langle X_i \rangle} \tag{3}$$

TRANSPORT IN RADIAL FLOW

We consider a fully penetrating well of radius r_w , recharging steadily a discharge Q per unit length. A plume of constant C_0 is injected through the well envelope A_0 during an

infinitesimal time Δt . At time t the plume is advected outwards and is distorted due to heterogeneity. Defining a radial front by $R = (1/M) \int r C dx$ and assuming ergodic conditions, from equation (3) we get $R(t) = \langle R \rangle = \langle X_r \rangle = \sqrt{Qt / \pi n R}$, where r stands for a radial coordinate such that the well is defined by $r = r_w$. In a similar vein, the rate of spreading of the plume around the front is characterized by $S_{rr}(t) = (1/M) \int (r - R)^2 C dx = X_{rr} + X_{drr}$. It is emphasized that the effect of spreading represented by $X_{rr} = \langle X_r'^2(t) \rangle$ is due entirely to the heterogeneous structure and it vanishes for $\sigma_y^2 = 0$, while X_{drr} , representing the effect of longitudinal pore-scale dispersion is generally negligible. The trajectory radial fluctuation is derived at first-order from the general equation (3), that becomes:

$$\frac{dX_r'}{dt} = u_r(R, 0, X_{d3}) + X_r' \frac{dU}{dR} \quad (4)$$

We limit the derivations to anisotropic formations (with the well normal to the bedding), with an anisotropy ratio, $e = I_v/I$, smaller than say 0.2. In line with Indelman & Dagan (1999), we maintain the effect of transverse pore-scale dispersion in the vertical direction only. The variance of the vertical pore-scale associated trajectory X_{d3} is evaluated approximately by $X_{d33} = \langle X_{d3}^2(t) \rangle = 2\alpha_{dT} \int_0^t U(t') dt' = 2\alpha_{dT} R$. Switching to R rather than t as the independent variable and integrating in equation (4) with $X_r' = 0$ for $R = 0$ (i.e. replacing the well by a singularity line), yields the basic result:

$$X_{rr}(R) = \langle X_r'^2(R) \rangle = U^2(R) \int_0^R \int_0^R \frac{u_{rr}(r', 0, X_{d3}(r'), r'', 0, X_{d3}(r''))}{U^2(r') U^2(r'')} dr' dr'' \quad (5)$$

Following Indelman & Dagan (1999) we use the approximation $u_{rr}(\mathbf{x}', \mathbf{x}'') \approx \sigma_y^2 \rho_y(\mathbf{x}' - \mathbf{x}'') U(r') U(r'')$, so that $X_{rr} \approx X_{rr}$ can be written with the aid of equation (5) as follows ($\hat{\rho}_y(\mathbf{k})$ is the Fourier transform of $\rho_y(\mathbf{x})$):

$$X_{rr}(R) = \frac{\sigma_y^2}{(2\pi)^{3/2} R^2} \int_0^R \int_0^R \int \int \hat{\rho}_y(\mathbf{k}) \exp[-ik_1(r' - r'') - k_3^2 \alpha_{dT} |r' - r''|] r' r'' dk dr' dr'' \quad (6)$$

For a Gaussian covariances $\rho_y(\mathbf{x})$, equation (6) yields:

$$X_{rr}^{(Gs)}(R) = \frac{\sigma_y^2 I^2}{3R^2} \int_0^{\bar{R}} \frac{(\bar{R} - r)^2 (2\bar{R} + r)}{\sqrt{1 + \pi\gamma r}} \exp(-\pi r^2 / 4) d\bar{r} \quad (7)$$

where $\gamma = \alpha_{dT} I / I_v^2 = (e^2 Pe)^{-1}$ with $Pe = I / \alpha_{dT}$ being the Peclet number and $\bar{R} = R / I$. A similar expression is obtained for exponential correlation. We have represented $X_{rr} / (\alpha_y^2 I^2)$ as a function of $\bar{R} = R / I$ for a few values of $\gamma = (e^2 Pe)^{-1}$ in Fig. 1 and for comparison $X_{rr} / (\alpha_y^2 I^2)$ for uniform flow and for $\gamma = 0$. It is seen that

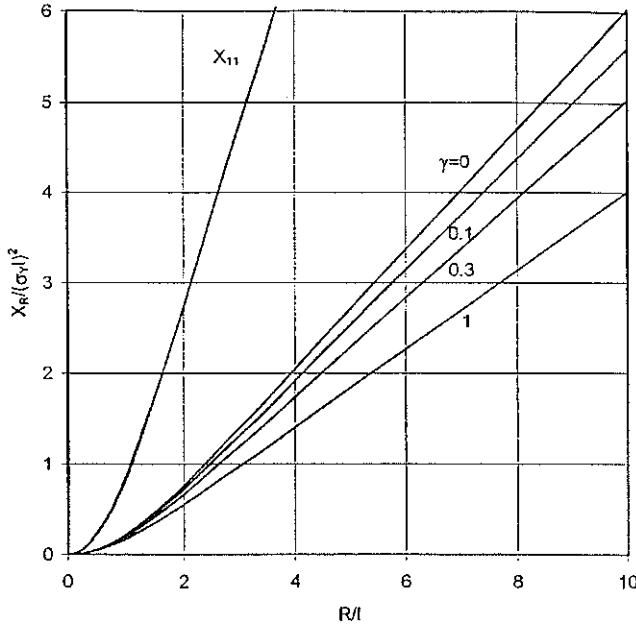


Fig. 1 Trajectory variance X_R as function of $R = Ut$.

the rate of the spreading due to heterogeneity is much smaller during radial flow than in uniform flow, for the same distance from the source.

The mean displacement R and its variance X_R define the motion of an ergodic plume and its spreading in terms of the centroid location and the second spatial moment. However, to derive the expression of the mean concentration $\langle C(x,t) \rangle$ with the aid of equation (2), one needs the complete pdf $f(X_R)$ of X_R . Under the first-order approximation X_R is normal and $\langle C \rangle$ satisfies the Focker-Planck equation:

$$\frac{\partial \langle C \rangle}{\partial t} + U(t) \frac{\partial \langle C \rangle}{\partial r} = D_R^{(app)}(t) \frac{\partial^2 \langle C \rangle}{\partial r^2}; \quad U = \frac{dR}{dt}; \quad D_R^{(app)}(t) = \frac{1}{2} \frac{dX_R}{dt} \quad (8)$$

where $D_R^{(app)}$ is defined as the apparent macrodispersion coefficient. Asymptotically, for $R/I \gg 1$, one can define an asymptotic constant apparent macrodispersivity $\alpha_R^{(app)} = D_R^{(app)}/U$ which is simply expressed in terms of γ for the exponential and Gaussian covariances:

$$\frac{6\pi\alpha_R^{(app)}(\gamma)}{\sigma_y^2 l} = \iint_{-\infty}^{\infty} \frac{\gamma p_3^2 dp_1 dp_3}{(p_1^2 + \gamma^2 p_3^4)(1 + p_1^2 + p_3^2)^{3/2}}, \quad \frac{3\alpha_R^{(app)}(\gamma)}{\sigma_y^2 I} = \int_0^{\infty} \frac{\exp(-\pi r^2/4)}{\sqrt{1 + \pi\gamma r}} dr \quad (9)$$

We have plotted $\alpha_R^{(app)}/(\sigma_y^2 I)$ as a function of γ in Fig. 2. The striking result is that $\alpha_R^{(app)}$ is smaller than the dispersivity α_L for uniform flow, by a factor of 3. In particular, for $Pe \gg 1$, i.e. for $\gamma \ll 1$, we get in equation (9) the simple result $\alpha_R^{(app)}(0)/(\sigma_y^2 I) = 1/3$ as compared with $\alpha_L(0)/(\sigma_y^2 I) = 1$ for uniform flow.

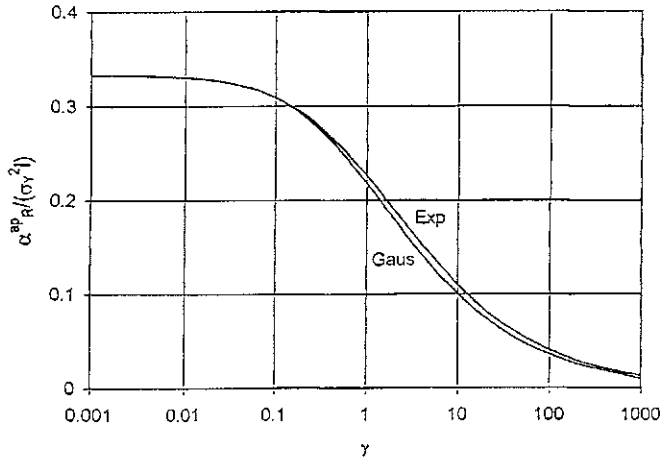


Fig. 2 Asymptotic apparent macro dispersivity as function of γ .

A related, but different coefficient, which we call the equivalent macrodispersivity, is defined as that pertaining to transport during radial flow in a homogeneous formation, such that the centroid velocity dR/dt and the rate of change of the second spatial moment of the plume $(1/2)dS_{rr}/dt$ are equal to those prevailing in the heterogeneous formation. In mathematical terms we seek the solution of:

$$\frac{\partial \langle C \rangle}{\partial t} + V(r) \frac{\partial \langle C \rangle}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left[\alpha_R^{(eq)} r V(r) \frac{\partial \langle C \rangle}{\partial r} \right]; \quad V = \frac{Q}{2\pi r} \tag{10}$$

for the initial condition of a pulse. Using the approximate solutions of equation (10) derived by Gelhar & Collins (1971) and by Dagan (1971), it can be shown that for large \bar{R} the rate of change of the second spatial moment $(1/2) dS_{rr}/dt = \alpha_R^{(eq)}/3$. Thus we arrive at the simple result that $\alpha_R^{(eq)} = \alpha_L$, i.e. the asymptotic equivalent macrodispersivity is equal to the one prevailing in uniform flow.

Figure 2 shows that for $\gamma < 0.01$ one can neglect the transverse mixing ($Pe = \infty$), i.e. pure advective transport of an inert tracer. By taking $\alpha_{dT} = 0$ in equation (6) we can express X_R in terms of a single quadrature as follows:

$$X_R(R) = \frac{2\sigma_Y^2 R}{3} \int_0^R \left(1 - \frac{3r}{2R} + \frac{r^3}{2R^3} \right) \rho_Y^h(r) dr \tag{11}$$

to be compared with $X_L = 2\sigma_Y^2 L \int_0^L (1 - x/L) \rho_Y(x) dx$ for uniform flow. One can easily calculate the trajectory variance (11) in particular cases. It follows from equation (11) that $\alpha_R^{(app)} = \lim_{R \rightarrow \infty} dX_R(R)/(2dR) = \sigma_Y^2 I/3 = \alpha_L/3$.

TRANSPORT IN DOUBLET FLOW

Next we consider transport of a conservative solute in flow driven by a recharging and a pumping well, neglecting the effect of pore-scale dispersion (see Dagan & Indelman

a pumping well, neglecting the effect of pore-scale dispersion (see Dagan & Indelman (1999), for the analysis of reactive solute transport). Flow is due to the head difference $2H_w$ between the two wells. The initial and boundary conditions for equation (1) with $D_d = 0$ are $C = 0$ for $t = 0$ and $C = C_0(t)$ at the recharging well envelope A_0 . We are interested in determining the flux-averaged concentration in the pumping well, defined by $C_f(t) = F(t)/Q(t)$, where $F(t) = n \int_A V_r(\mathbf{x})C(\mathbf{x},t)dx$ is the solute flux and

$Q_f = n \int_A V_r(\mathbf{x}) dx$ is the water flux. Thus, C_f is the time-dependent concentration measured in the pumping well if complete mixing takes place.

Let $g_\nu(\tau; \mathbf{b})$ be the pdf of the travel time τ needed for a fluid particle, originating at location $\mathbf{x} = \mathbf{b}$ on the envelope of the recharging well, to reach the pumping well (Dagan & Indelman, 1999). Then, it can be shown that

$$C_f(t) = 1/A_0 \int_{A_0} \int_0^t C_0(t_0) g_\nu(t - t_0; \mathbf{b}) dt_0 d\mathbf{b}. \quad (11)$$

Adopting the approach employed for transport in radial flows in highly anisotropic formations, in the previous Section, we arrive (Dagan & Indelman, 1999) at the following expressions for the mean and variance of the travel time τ :

$$\tau^{(0)}(\theta) = \frac{\pi n l^2}{Q} \frac{1 - \theta \operatorname{ctg} \theta}{\sin^2 \theta}, \quad \sigma_r^2(\theta) = \frac{Q^2 \sigma_V^2}{4\pi^2 n^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{[\mathbf{r}(\phi', \theta) - \mathbf{r}(\phi'', \theta)]^2}{U^2(\phi', \theta) U^2(\phi'', \theta)} d\phi' d\phi'' \quad (12)$$

Here l is the distance between the wells, Q is the wells discharge, $U(x_1, x_2)$ is the mean flow velocity and $\mathbf{x} = \mathbf{r}$ is the mean trajectory of a fluid particle. The latter are given in a parametric form by:

$$x_1 = \frac{l \sinh \phi}{2 \cosh \phi + \cos \theta}, \quad x_2 = \frac{l \sin \theta}{2 \cosh \phi + \cos \theta}, \quad U = \frac{Q}{\pi n l} (\cosh \phi + \cos \theta) \quad (13)$$

Here the equation $\theta = \text{const}$ defines streamlines, the shortest and longest paths corresponding to $\theta = 0$ and π , respectively, while ϕ varies within $(-\infty, \infty)$.

To evaluate the mean concentration in the pumping well one needs the joint pdf $g(V_{r0}, \tau)$ of the travel time τ and the radial velocity V_{r0} at the recharging well envelope. We adopted a lognormal pdf for τ (Cvetkovic *et al.*, 1996) and calculate the mean and variance of τ using equation (12). Figure 3 shows the dependence of C_f on dimensionless time $t' = Qt/(\pi n l^2)$ for the case of a pulse of short duration, i.e. $C_0(t) = \delta(t)$. The continuous lines correspond to $\sigma_V^2 = 0.5$ and different values of $l' = l/I = 0.01, 1, 4$ and 100 , whereas a dashed line shows C_f in the homogeneous medium.

APPLICATION OF RESULTS

The above developments constitute the basis of a few possible applications. The one we wish to discuss here is identification of aquifer transport properties by a single recharging well or by a doublet. The idea of injecting a tracer pulse through a well and

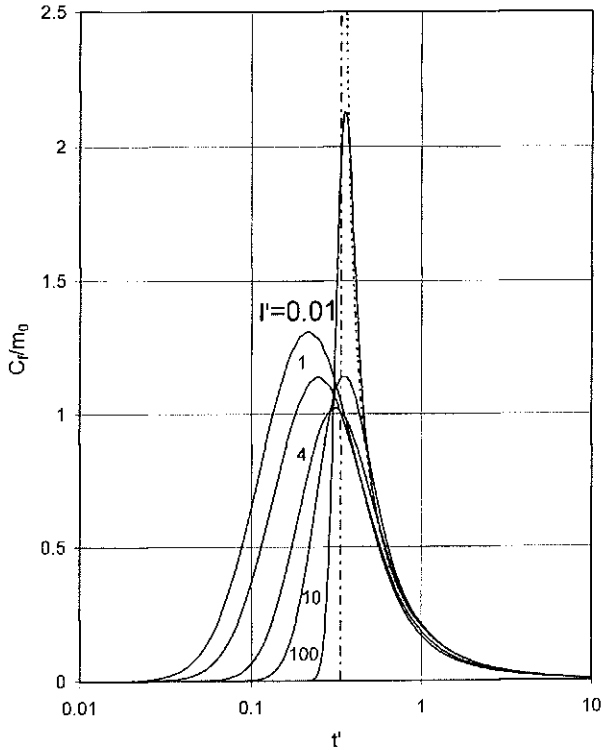


Fig. 3 Flux averaged concentration C_f at the pumping well for instantaneous injection.

well was considered a long time ago (for recent experimental field work see e.g. Ptak & Teutsch, 1994). The advantage of such a test as compared with a similar natural gradient flow experiment is its simplicity and the better control of the flow pattern as well as the shorter duration. However, in the past the interpretation of results was carried out using the assumption of an homogeneous aquifer or a simple layered structure. Since most aquifers have a spatially variable permeability of anisotropic structures, the present results offer the possibility of identifying the parameters characterizing the structure, e.g. σ_V^2 and I , by a best fit of the measured concentration and the theoretical one, as illustrated in Fig. 3. We hope that such applications will be undertaken in the future.

Acknowledgements The support of the Israel Ministry of Science and Technology and of the French Ministère de l'Éducation, de l'Enseignement Supérieur et de la Recherche, is kindly acknowledged.

REFERENCES

- Chen, C. S. (1987) Analytical solution for radial dispersion with Cauchy Boundary at injection well. *Wat. Resour. Res.* **23**, 1217–1224.
- Cvetkovic, V., Cheng, H. & Wen, X. H. (1996) Analysis of nonlinear effects on advective tracer transport in heterogeneous aquifers using the Lagrangian statistics of travel time. *Wat. Resour. Res.* **32**, 1671–1680.

- Dagan, G. (1971) Perturbation solutions of the dispersion equation in porous mediums. *Wat. Resour. Res.* **7**, 135–142.
- Dagan, G. (1989) *Flow and Transport in Porous Formations*. Springer-Verlag, Heidelberg, Germany.
- Dagan, G. & Indelman, P. (1999) Reactive solute transport in flow between a recharging and a pumping well in a heterogeneous aquifer. *Wat. Resour. Res.* **35**(12), 3639–3647.
- Gelhar, L. W. & Collins, M. A. (1971) General analysis of longitudinal dispersion in nonuniform flow. *Wat. Resour. Res.* **7**, 1511–1521.
- Indelman, P., Fiori, A. & Dagan, G. (1996) Steady flow toward wells in heterogeneous formations: mean head and equivalent conductivity. *Wat. Resour. Res.* **32**, 1975–1983.
- Indelman, P. & Dagan, G. (1999) Solute transport in divergent radial flow through heterogeneous porous media. *J. Fluid Mechanics* **384**, 159–182.
- Ptak, T. & Teutsch, G. (1994) Forced and natural tracer tests in a highly heterogeneous porous aquifer: instrumentation and measurements. *J. Hydrol.* **159**, 79–104.