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Modelling of karst development considering conduit–matrix exchange flow

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Abstract This paper presents a numerical model study simulating the early karstification of a single conduit embedded in a carbonate matrix system. A hybrid continuum–discrete pipe flow model (CAVE) is used for the modelling. The effects of the coupling of the two flow systems on the type and duration of karstification is studied for different initial diameters of the conduit. Assuming a linear exchange term for the coupling of the conduit and matrix system leads to more rapid development of the conduit in the scenario presented. This effect is most pronounced for small initial diameters, where the rate of karstification is increased by a factor of 15 compared to the case with no exchange flow to the matrix system.

INTRODUCTION

Karst groundwaters are highly vulnerable due to the fast transport of pollutants in complex drainage systems. In order to facilitate groundwater risk assessment it is important to obtain information on the hydraulic properties of the conduit network, which is the main factor controlling the spreading of contaminants. In particular, the unknown structure of the solutionally developed preferential flowpaths in the carbonate rock has to be clarified for the development of remediation strategies. One approach to characterizing a karst aquifer is through understanding the genesis of the complex hydraulic structures incorporated in it.

MODEL STRUCTURE

In this work a well accepted conceptual model of a karst aquifer is used. The flow system of a karst aquifer is assumed to consist of a conduit system, characterized by low storage and high hydraulic conductivity, and a fissured system (matrix) with high storage and low conductivity. Groundwater flow in the fissured system is modelled through a continuum approach using Boussinesq's equation. Flow in the conduit system, represented by a network of cylindrical tubes intersecting at nodes, is governed by Kirchhoff's law which states that total inflow and total outflow balance at each node. The relation between hydraulic head and discharge is adapted to the flow condition, i.e. the model accounts for laminar and turbulent flow. Exchange between the fissured and the conduit system is modelled by a linear steady state exchange term, i.e. exchange flux is proportional to the head difference between the two flow systems (Barenblatt *et al.*, 1960).

Dissolution of calcite from the tube walls leads to enlargement of the tubes and thus a higher conductivity of the conduit system. According to Dreybrodt (1990) calcite dissolution can be described by a first-order rate law far from saturation (fast kinetics) and a fourth-order rate law near saturation (slow kinetics), which becomes active at a relative saturation greater than 0.9. Transport of dissolved calcium ions in the tubes is described by a one-dimensional advection equation, assuming instantaneous and complete mixing at the tube intersections. These processes have been implemented in the modelling tool CAVE (Carbonate Aquifer Void Evolution), which can be used to study evolution of karst features. For a more detailed model description see Clemens *et al.* (1996).

Determination of the exchange coefficient for the linear steady state exchange flow is only possible by assuming additional geometrical information for the inter-conduit blocks. Approaches by Barenblatt *et al.* (1960), Warren & Root (1963), Narasimhan (1984) and Lei (1999), have in common the feature that exchange flux is determined by the difference of hydraulic head between the matrix system and the conduit system (Δh), the hydraulic conductivity of the matrix system (K_m), the exchange surface between the conduit and the matrix (A_F), and some geometry factor (α), which is dependent on conduit geometry, e.g. fracture spacing:

$$Q_{ex} = \alpha\, A_F\, K_m\, \Delta h = \alpha_0\, \Delta h \tag{1}$$

where α_0 is termed the exchange coefficient and has units of $m^2\ s^{-1}$.

Exchange flow is mainly determined by the hydraulic conductivity of the matrix, which gives the order of magnitude for the exchange coefficient. The other factors for the exchange coefficient are determined by the fracture/block geometry, which is not known, and for the description of which parameters have to be assumed. An uncertainty of one order of magnitude is therefore likely for the exchange coefficient and has to be accounted for during modelling.

To study the effect of matrix exchange on karstification, a simplified scenario is used (Fig. 1). A single conduit of 500 m length, consisting of 50 tubes of 10 m each, is

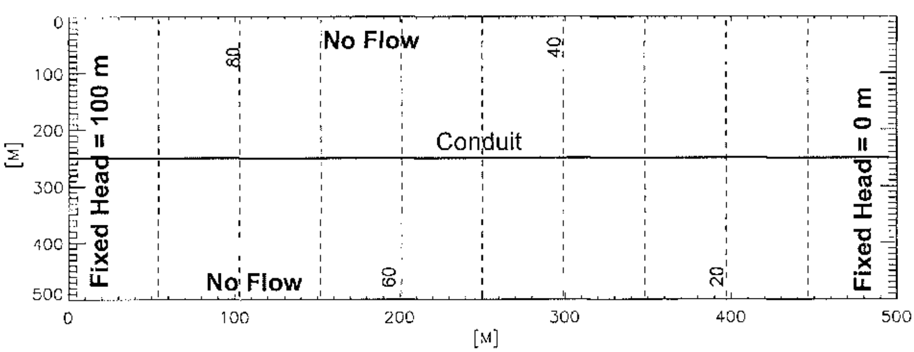

Fig. 1 Model scenario showing the single conduit embedded in a matrix system. Dashed lines are piezometric heads in the matrix system at the beginning of the simulation.

placed within a matrix system of 500 m length and 500 m width. The conduit represents a solutionally enlargeable percolating pathway in the limestone, that may consist of many fractures, joints or a bedding plane. Conduit boundary conditions are a fixed head of 100 m at the inflow end and a fixed head of 0 m at the outflow end. Matrix boundary conditions are a fixed head of 100 m at the top end, of 0 m at the lower end and no flow at the sides and the bottom of the matrix system. The hydraulic conductivity of the matrix system is 1×10^{-6} m s^{-1}, the aquifer is assumed confined. Calcium equilibrium concentration is 2 mmol l^{-1}. Water entering the conduit at the inflow end has a calcium concentration of 0 mmol l^{-1}, while water entering the conduit from the matrix system has equilibrium concentration. The fast first-order kinetic rate constants for laminar and turbulent flow are 2.5×10^{-5} cm s^{-1} and 5×10^{-5} cm s^{-1}, respectively. The kinetic constant for the slow fourth-order dissolution, active if calcium concentrations are above 90% of the equilibrium concentration, is 1.3×10^{-13} cm^{10} mol^{-3} s^{-1} for both laminar and turbulent flow. This scenario is typical for early karstification, where only a part of the total flow is through the conduit system and thus the fixed head boundary is realistic. In the following the effects of variations in exchange coefficient and initial diameter on the development of a karst conduit are studied in more detail. Two cases are considered: without and with matrix exchange. If matrix exchange is accounted for, the exchange coefficient is varied from 1×10^{-8} m^2 s^{-1} to 1×10^{-4} m^2 s^{-1}, because matrix conductivity is 1×10^{-6} m s^{-1}. Initial diameters are varied from 5×10^{-5} m to 5×10^{-4} m, which is in the typical range for fractures at greater depth.

RESULTS

On the left hand side of Fig. 2 the temporal development of the conduit for the case of no matrix exchange is shown. Conduit diameters are enlarged only slowly at first, but uniformly along the conduit (Fig. 2(a)). This is because slow, fourth-order kinetics are active along the whole conduit (saturation > 0.90) and flow rates are low. Flow rates are governed by the smallest diameters at the conduit outflow and are constant along the conduit (Fig. 2(b)). After 4000 years, conduit diameters have been widened to more than 1 mm at the outflow end, and to more than 100 mm at the inflow end, due to fast first-order kinetics propagating into the conduit (Fig. 2(c)). The range where first-order kinetics prevails now propagates quickly through the conduit and reaches the outflow end only 400 years later. Turbulent flow becomes active, first at the outflow end of the conduit, then along the whole conduit length, and the conduit is widened uniformly with fast first-order turbulent kinetics. Flow rates increase dramatically.

Development of the conduit in the initial phase is governed by slow fourth-order kinetics until the range of fast first-order kinetics has propagated through the conduit and completely dominates further enlargement. This time period is termed the "breakthrough time" (Dreybrodt, 1990), because immediately after breakthrough there is an enormous increase in flow rates and further dissolution is governed by fast first-order kinetics. Breakthrough time in the case of no matrix exchange is 4400 years for the scenario studied here (Fig. 2(c)).

The right hand side of Fig. 2 shows the development of the conduit if matrix exchange is allowed for. Conduit development is much quicker than without

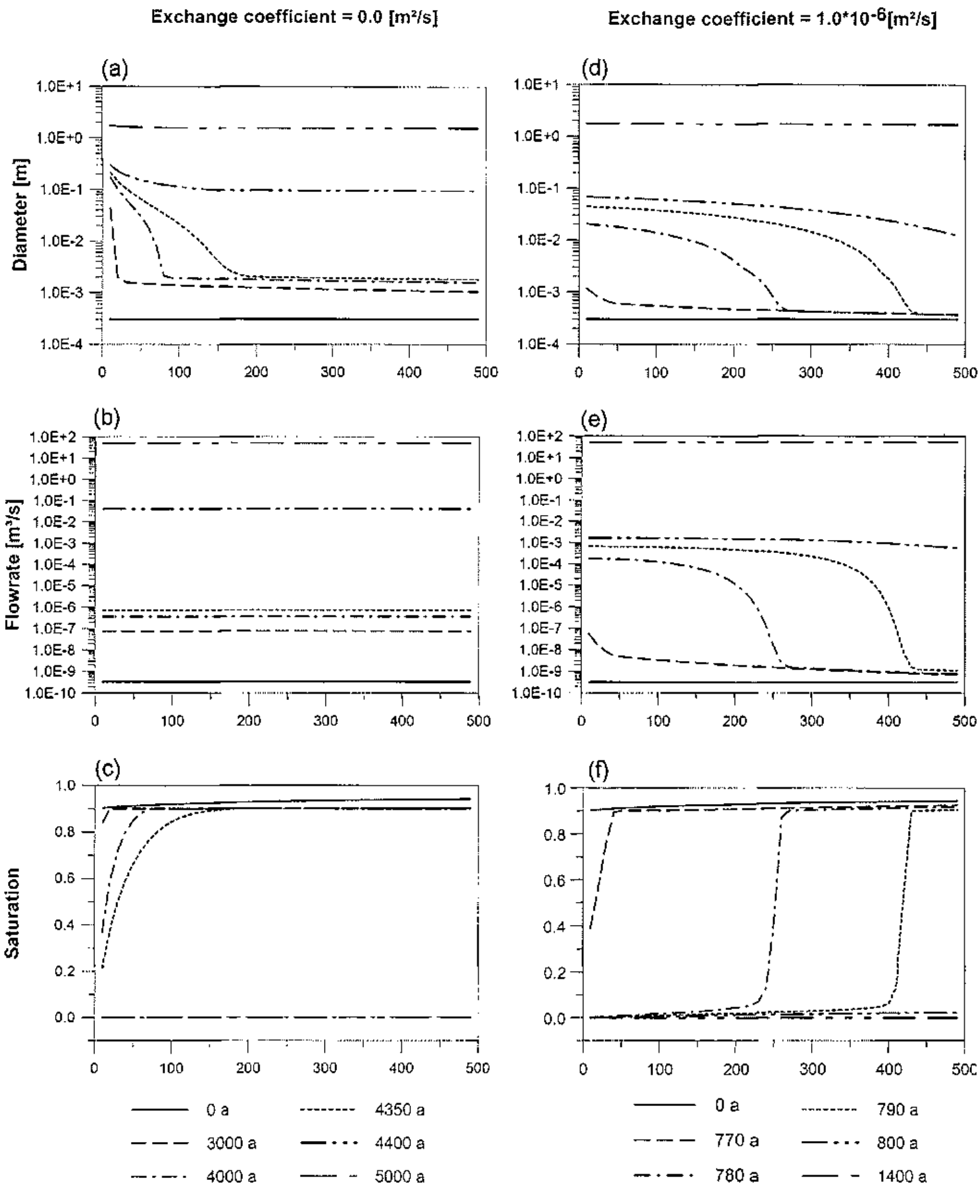

Fig. 2 Time development of diameter, flow rate and saturation of a conduit for the case of no matrix exchange (left side, (a) to (c)) and the case of matrix exchange using an exchange coefficient of 1×10^{-6} m^2 s^{-1} (right side, (d) to (f)). Direction of flow is from the left to the right.

matrix exchange and breakthrough time is 800 years only. Conduit diameters are widened more evenly along the conduit (Fig. 2(d)), while flow rates differ along the conduit (Fig. 2(e)).

If the inflow end of the conduit has been somewhat enlarged, the hydraulic head in the conduit just downstream of the inflow end is higher than in the matrix and matrix exchange flow is directed into the matrix. As a result, water with a high calcium concentration and low aggressiveness is directed into the matrix and water with a low calcium concentration (highly aggressive water) is re-supplied to the conduit from the fixed head boundary. Therefore matrix exchange provides an additional sink to water high in calcium and, as a result, acts as a mechanism for the deep penetration of aggressive water. With no matrix exchange, flow through the conduit is limited by the smallest diameter at the outflow end of the conduit, while flow rates at the inflow end of the conduit are much higher than at the outflow end, if matrix exchange is accounted for (Fig. 2(e)). The region of high matrix infiltration propagates downstream along the conduit until it reaches the outflow end of the conduit at the time of breakthrough. In this region conduit flow rates and diameters decrease along the conduit by orders of magnitude, while saturation increases to more than 0.9 and kinetics switch from fast first-order to slow fourth-order (Fig. 2(f)).

Since the exchange coefficient can only be calculated to within an order of magnitude, it was varied to determine the effect of this uncertainty. Figure 3 shows the dependence of breakthrough times on initial diameter for different exchange coefficients. For all diameters, breakthrough times are highest if no matrix exchange is assumed. For exchange coefficients varying from 1×10^{-4} m^2 s^{-1} to 1×10^{-8} m^2 s^{-1} breakthrough times are similar. This shows that the exact value of the exchange coefficient does not need to be known, as long as the order of magnitude can be obtained. Breakthrough times are much smaller if matrix exchange is considered.

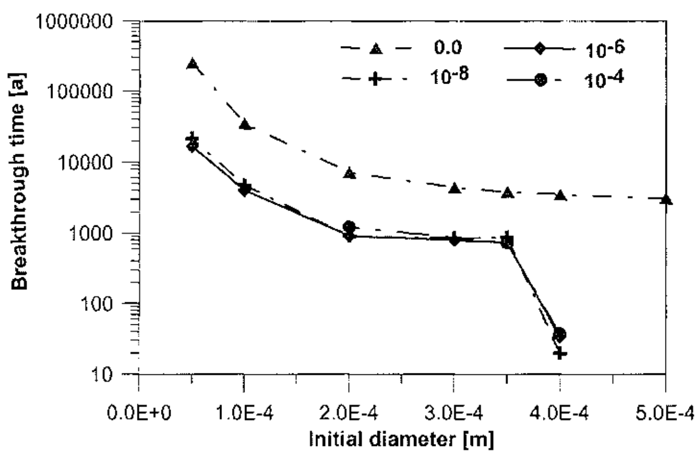

Fig. 3 Breakthrough times for different initial diameters and varying exchange coefficients (units of m^2 s^{-1}).

While the breakthrough time without matrix exchange is five times larger than the breakthrough time with matrix exchange at an initial diameter of 3.5×10^{-4} m, the breakthrough time is already 15 times longer at 5×10^{-5} m.

Using an initial diameter of 4×10^{-4} m, the breakthrough time is only about 20 years, if matrix exchange is considered. This is because under the boundary conditions applied, breakthrough could have happened at smaller diameters, i.e. the initial diameter of the conduit is too large. This can be seen from Fig. 2(d) where the conduit diameter at the outflow end of the conduit has grown to 3.7×10^{-4} m at breakthrough, which is smaller than 4×10^{-4} m. Therefore, there exists a starting initial diameter which is so large that breakthrough is instantaneous. This effect also occurs in the case of no matrix exchange; the initial diameter is then 2×10^{-3} m.

DISCUSSION AND CONCLUSION

The impact of matrix exchange on karstification has been investigated for a simplified scenario with a single conduit in a karst catchment area. If matrix exchange is accounted for, karstification of small conduits with initial diameters of the order of 10^{-5} m to some 10^{-4} m is faster by a factor of 5 to 15 than without matrix exchange. This is especially important in the case of hydraulic structures, such as reservoirs or dams, because the lifetimes of these structures can thus be significantly shortened (Bauer *et al.*, 1999).

The applied fixed head boundary is valid only until breakthrough, because afterwards flow rates increase quickly to very high values, which are not typically supported by natural conditions. The scenario presented here is therefore valid only for the initial stages of karstification, when flow rates are low. If flow rates become too high during the simulation, the fixed head boundary should be switched to a fixed flow boundary, to represent the limited water supply.

A dam is able to maintain the high head at the inflow end of the conduit until breakthrough and for some time afterwards. If water supply to the inflow end of the conduit is limited, i.e. by recharge, this high head may not be maintained throughout conduit development since flow rates at the inflow end may exceed water supply. In this case, more typical for late stages of karstification, different boundary conditions have to be specified. A flux-limited boundary condition will be implemented in the CAVE model to enable the study of karstification under such boundary conditions.

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