Integrating tracer with remote sensing techniques for determining dispersion coefficients of the Dâmbovita River, Romania

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Abstract Knowledge of dispersion coefficients in longitudinal, lateral and vertical flow directions is of utmost importance when evaluating the time–concentration distribution of pollutants at any point in a stream. Remotely sensed field data (from the National Administration of the Ocean and Atmosphere and teledetection) were used to identify the river sector for a dye tracing experiment. The main criteria were to locate a sector without significant anthropogenic influence and with similar hydraulic conditions. The sector selected for sampling was 2.5 km long. The longitudinal and three-dimensional dispersion coefficients were determined. The time–concentration curve for rhodamine dye obtained under different hydraulic conditions of flow (different stages) was measured on the Dâmbovita River. The stage was nearly constant for each experiment but different in each case, therefore it was possible to obtain information for different regimes. Additional information needed to make use of the one-dimensional and three-dimensional mathematical models of dispersion was obtained at the Malu cu Flori gauging station (continuos stage recording and stage–discharge curve). Finally, the experimental data were used to verify the mathematical dispersion model proposed for the Dâmbovita River.

INTRODUCTION

Stream dispersion phenomena, dilution and transport of pollutants in a channel is caused by advective and diffusive transport mechanisms. The range of the dispersion coefficients in the longitudinal, lateral and vertical directions of flow is very important in evaluating the time–concentration distribution of a pollutant at any point in a stream. Although significant advances have been made in understanding and modelling dispersion in natural streams, the problem of dispersion coefficients based on field and experimental data needs further study. The main aim of the experiment was to determine the dispersion coefficients and to verify the predicted concentration by the proposed mathematical model.

The remotely sensed data from the National Administration of the Ocean and Atmosphere were useful in identifying the river sector. However the spatial resolution used for dispersion was inadequate for mapping the river. Additional teledetection data had also to be used.

In the first section of the paper, the dispersion model is presented; in the second, both the experiment and the resulting data are shown.

MATHEMATICAL MODEL OF DISPERSION

The concentration distribution of a dye in a turbulent stream is governed by an equation based on the law of conservation of mass (Landau & Lifchitz, 1971). In the case of no other sinks and sources in the reach that is:

$$\frac{\partial \bar{c}}{\partial t}+\bar{v}_i\frac{\partial \bar{c}}{\partial x_i}=\frac{\partial}{\partial x_i}\left[D_{ij}\frac{\partial \bar{c}}{\partial x_j}+D\frac{\partial \bar{c}}{\partial x_i}\right] \tag{1}$$

$\bar{c}$ is the mean local concentration of dye (parts per billion by volume), $\bar{v}$ is the mean velocity (i refers to Cartesian coordinates x, y, z considered in the direction of flow, transversal, along it and vertically down, respectively), D_{ij} and D are the turbulent and molecular diffusion coefficients; v_x is the longitudinal velocity. The diffusion and convective terms are neglected due to the mean velocity components in the y and z directions (because they are usually small compared with the v_x term). Considering $D_i = (D_x, D_y, D_z)$, equation (1) becomes:

$$\frac{\partial c}{\partial t}+\bar{v}_i\frac{\partial c}{\partial x_i}=D_i\frac{\partial^2 c}{\partial x_i \partial x_i} \tag{2}$$

The initial condition is:

$$c(x, y, z) = 0 \text{ for } x > 0, y > 0 \text{ and } z > 0 \tag{3}$$

The boundary conditions are:

$$c(x, y, z, t) = 0 \text{ for all values of } t$$

$$\frac{\partial c}{\partial x}\rightarrow 0 \text{ as } x\rightarrow \propto; \quad \frac{\partial c}{\partial y}\rightarrow 0 \text{ as } y\rightarrow \propto \text{ (physically } y = B/2)$$

$$\frac{\partial c}{\partial z}\rightarrow 0 \text{ as } z\rightarrow \propto \text{ (physically } z = H)$$

Assuming $D_x = D$ and mean velocity $v_x = V$ for the reach, the solution of equation (2) could be written:

$$c(x,y,z,t)=\frac{M}{\sqrt{4\pi Dt}}\exp\left[-\frac{(x-Vt)^2}{4Dt}\right]\frac{2}{\sqrt{4\pi Dt}}\exp\left[-\frac{y^2}{4D_y t}\right]\frac{2}{\sqrt{4\pi D_z t}}\exp\left[-\frac{z^2}{4D_z t}\right] \tag{4}$$

which does not satisfy the first boundary condition, but:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}c(x,y,z,t)\mathrm{d}x\mathrm{d}y\mathrm{d}z = M \tag{5}$$

To support the physical meaning of the river, this can be approximated as:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}c(x,y,z,t)\mathrm{d}x\mathrm{d}y\mathrm{d}z = M -$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}c(x,y,z,t)\mathrm{d}x\mathrm{d}y\mathrm{d}z-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}c(x,y,z,t)\mathrm{d}x\mathrm{d}y\mathrm{d}z-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}c(x,y,z,t)\mathrm{d}x\mathrm{d}y\mathrm{d}z \tag{6}$$

which indicates that for natural flow conditions in a stream, with finite values of B (the surface width) and H (the depth of flow), the amount of dye recovered would be less than the total amount of dye injected. The amount of dye lost or detained in dead zones laterally and vertically is assumed to be given by the three quantities on the right-hand side of the equation. Equation (3) is valid for a short interval of time because it does not take into account the amount of dye that may be lost due to absorption and decay. For downstream distances far from the injection point, equation (3) may be necessary to account for such a loss. Equation (3), known as the three-dimensional mathematical model of dispersion, has been used to determine the vertical and lateral dispersion coefficients for different reaches of the stream where the measured time–concentration curves of dye are available. The value of D_x is assumed to be equal to the value of longitudinal dispersion D, determined from the one-dimensional mathematical model of dispersion.

Assuming that mixing is uniform at the measurement cross-section, the one-dimensional case with a constant longitudinal dispersion coefficient ($D_{xx} = D$), the equation for the mean concentration obtained from equation (1) following the standard procedure of averaging is:

$$\frac{\partial c}{\partial t} + \bar{v}_i \frac{\partial c}{\partial x_i} = D \frac{\partial^2 c}{\partial x_i \partial x_i} \tag{7}$$

where $\bar{v}_i = V$ is the average velocity of flow at a section which is assumed to be independent of time.

$$c(x,t) = \frac{M}{A\sqrt{4\pi Dt}} \exp\left[-\frac{(x - Vt)^2}{4Dt}\right] \tag{8}$$

It may be observed that D_x may not be the same as D, but for larger values of either x or t, $D_x \cong D$.

The lateral dispersion coefficient D_y is expected to depend on the vertical dispersion coefficient D_z; of B, H, and V and the mean velocity of flow at the sampling station, V_s. A dimensional analysis shows that (Bansal, 1981):

$$D_y = \left(\frac{V_s}{V}\right)^2 \left(\frac{B}{H}\right)^2 (D_z) \tag{9}$$

in which $V_s = Q/A$, $V = x/t_p$ and Q is the discharge ($m^3\ s^{-1}$); A is the area of the cross-section (m^2); x is the distance from the point of injection (m); and t_p is the travel time (h). D_z is determined from the best fit between the computed and the measured time-concentration curves. For a value of D_z the concentration distribution can be computed from the three-dimensional mathematical model of dispersion.

Adaptation of the mathematical model of dispersion to the river conditions

The loss of solute concentration increases downstream as the distance x increases. Using the linearity of the problem, a modified solution of equation (7) first suggested is:

$$c(x,t) = \frac{x}{Vt} \frac{M}{A\sqrt{4\pi Dt}} \exp\left[-\frac{(x - Vt)^2}{4Dt}\right] \tag{10}$$

which appears to take the dye loss into account, as the term is linear of x. But in natural streams an amount of dye is lost due to photochemical decay and benthic adsorption by vegetation and suspended solids present in the stream, as well as by retention in the dead areas. So (10) is adjusted to take this into account, giving:

$$c(x,t)=\frac{1}{K}\left(1-K_0\frac{x}{Vt}\right)\frac{M}{A\sqrt{4\pi Dt}}\exp\left[-\frac{(x-Vt)^2}{4Dt}\right] \tag{11}$$

in which K is a regional dispersion factor defined for a stream and K_0 is a coefficient of loss of dye in a reach. K was determined by trial and error by comparing the computed and observed time-concentration curves for the stream and K_0 was derived from a dimensional analysis of the dependent factors:

$$K_0=\frac{1}{4}\left(\frac{VB}{4D}+\frac{\log(2H/L)}{\log(Dt_p/L^2)}+K_1\right) \tag{12}$$

$VB/4D$ is a loss factor accounting for the detention of dye in lateral dead zones; $\log(2H/L)/\log(Dt_p/L^2)$ is a loss factor accounting for the detention of dye in the vertical dead zones; $K_1 \approx 0.5$ accounts for the loss of dye due to decay and adsorption which is relatively constant in streams. Finally, the one-dimensional mathematical model of dispersion in a stream can be described by:

$$c(x,t)=\frac{1}{K}\left(1-\frac{xB}{16Dt}-\frac{x}{4Vt}\frac{\log(2H/L)}{\log(Dt_p/L^2)}\right)\frac{M}{A\sqrt{4\pi Dt}}\exp\left[-\frac{(x-Vt)^2}{4Dt}\right] \tag{13}$$

This is a linear solution of equation (7) that satisfies all the necessary initial and boundary conditions required in the case of a natural stream. The value of D, which

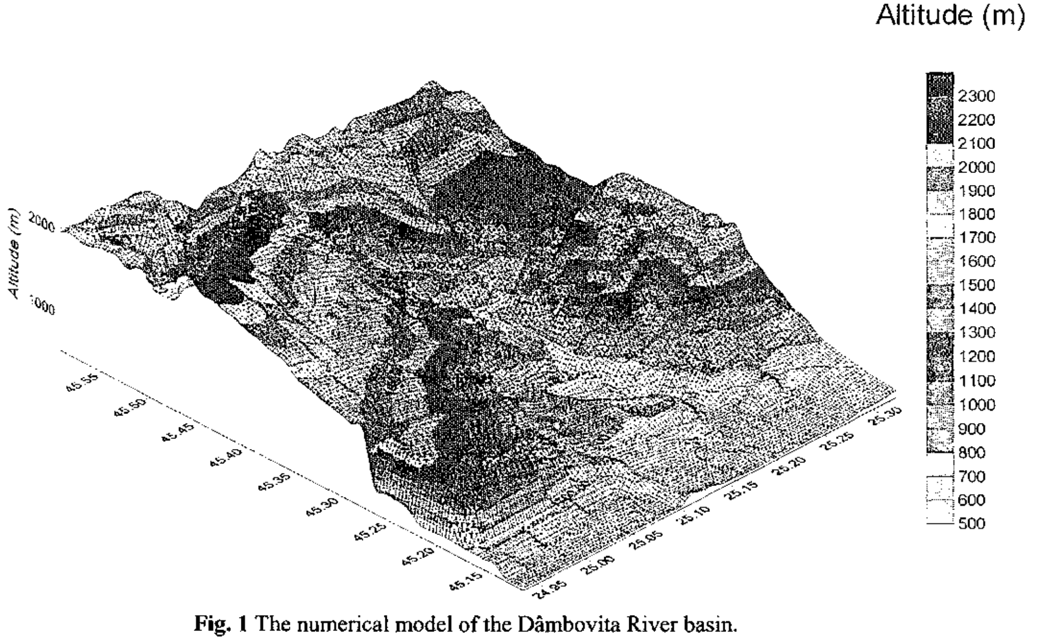

Fig. 1 The numerical model of the Dâmbovita River basin.

gives the best fit between the computed and measured time-concentration curves, is accepted as an appropriate value for the reach.

EXPERIMENTAL DATA

Using a local image of the basin, a 2.5 km section of the Dâmbovita River was chosen for the experiment. The digital elevation model of the river basin is presented in Fig. 1. The additional information needed to make use of the one-dimensional and three-dimensional mathematical models of dispersion was obtained at the Malu cu Flori gauging station on the Dâmbovita River.

We conducted tracer experiments on the Dâmbovita River in 1995 (Q = 12.9 $m^3 s^{-1}$) and 1997 (Q = 7.88 $m^3 s^{-1}$). Rhodamine B dye was injected instantaneously into the river. Concentration distributions were measured at four cross-sections downstream from the point of injection. The concentrations for each section were measured using a fluorometer. Gaspar & Oreaeanu (1987) presented the method.

Incomplete mixing over the width of the river is very important. Therefore all samples were taken simultaneously at three sites at each cross section of the river. In the last control section full mixing was assumed and dye concentrations were measured using a single fluorometer.

The dispersion coefficients were directly proportional to the breakthrough. Greater dispersion coefficients were reflected by decreased peak breakthrough concentrations as the tracer migrated downstream (Figs 2 and 3).

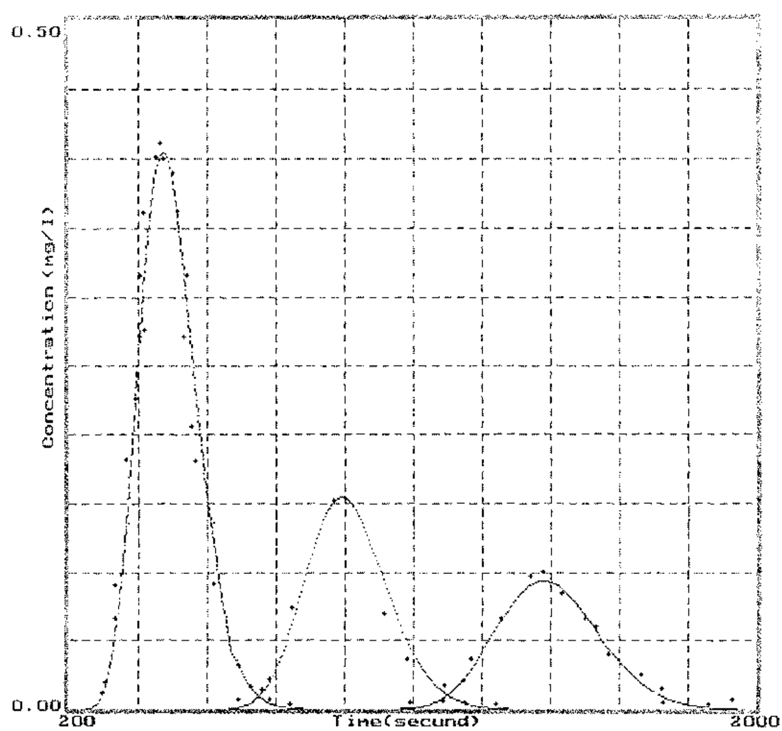

Fig. 2 Tracer-response curve for stream sections in 1995.

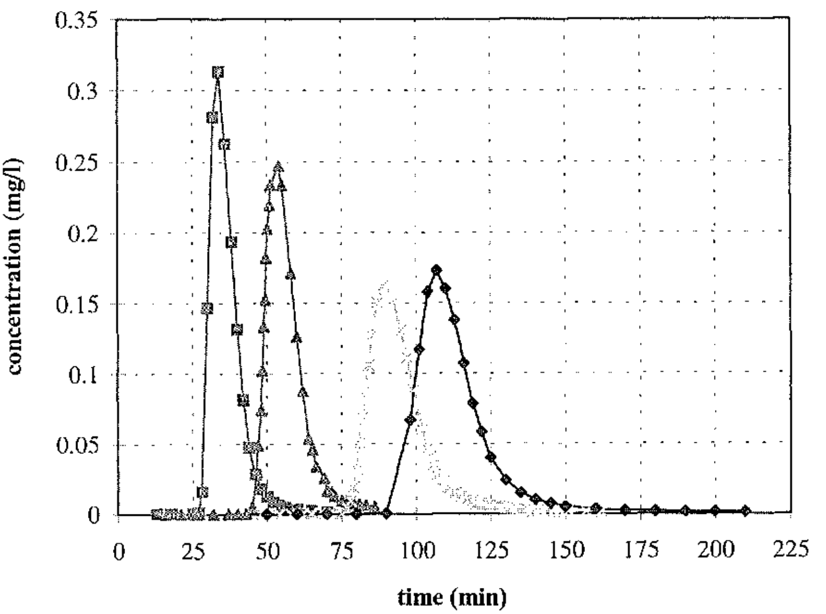

Fig. 3 Tracer-response curve for stream sections in 1997.

VERIFICATION OF THE MATHEMATICAL DISPERSION MODEL

To verify the correctness of the longitudinal dispersion coefficient as predicted from the one-dimensional model, a test was required to check the response of the model at different locations along the stream (Fig. 4). The computed curve is obtained by determining D, K, and K_1 in the first section of the experiment; these values are used to calculate the peak concentration by equation (13). Figure 4 shows that the deviation is not significant and the check proves that the one-dimensional model of dispersion functioned satisfactorily.

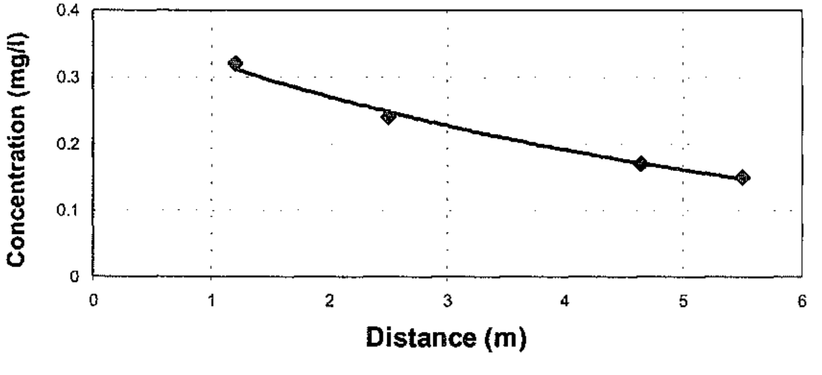

Fig. 4 Computed and observed peak concentrations for the Dâmbovita River.

The experiment allowed us to have some idea of the degree of magnitude of the coefficients of dispersion. In the Malu cu Flori gauging station section, the values obtained were: $D_x = D = 20$ m^2 s^{-1}, for $Q = 12.9$ m^3 s^{-1} ($K = 0.40$ and $K_1 = 0$) and $D_x = D = 11.5$ m^2 s^{-1}, $D_y = 0.085$ m^2 s^{-1}, $D_z = 0.000\,06$ m^2 s^{-1} for $Q = 7.88$ m^3 s^{-1} ($K = 0.27$, $K_1 = 0$).

The results of the three-dimensional model of dispersion for the 1997 experiment, which permitted us to identify all the components of the dispersion vector for one section, are shown in Fig. 5.

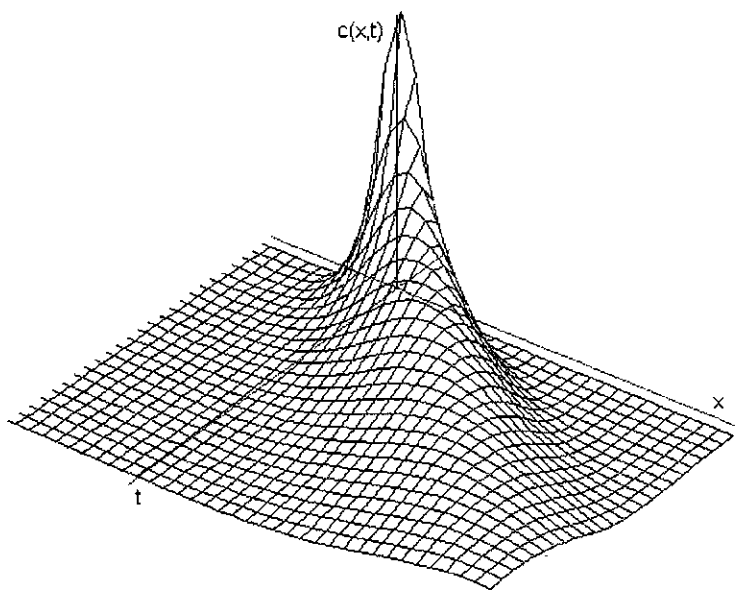

Fig. 5 Three-dimensional dispersion model of the Dâmbovita River.

CONCLUSION

The predicted values of *D* agree well with the dispersion characteristics of the stream. The values are dependent on the flow regime of the river. In conclusion, one experiment is not sufficient to determine the coefficient of dispersion, but a relationship depending of the river regime and the hydraulic conditions of flow has to be obtained.

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