

Variability in spectral characteristics of hydrologic data

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Abstract Conventionally, hydrologic time series have been studied in both the time and frequency domains. However, neither the time domain nor the frequency domain is adequate for revealing the underlying structure of such time series. Hydrologic time series can be shown to consist of a stochastic component with a continuous spectrum in addition to a number of added components with discrete spectra. These added components are often interpreted as the effect of a number of natural phenomena such as the sunspot activity, El Niño Southern Oscillations (ENSO), Quasi-Biennial Oscillations (QBO) and others. However, these components are usually not well defined in hydrologic time series. This study investigates the variability in both the frequency and amplitude of the cyclic components in hydrologic time series. For this purpose, the Multi-Taper spectral analysis method (MTM) (Thomson, 1982) coupled with evolutionary spectral analysis (Priestley, 1965) are used to analyse a number of rainfall, flow, and temperature time series from the Midwest region of the United States. The results from evolutionary spectral analysis indicate variability in both the frequency and magnitude of the cyclic components in the studied time series. Furthermore, a statistical test of stationarity indicates that the continuous part of the spectrum varies significantly with time, indicating nonstationarity of the stochastic processes producing these hydrologic time series.

INTRODUCTION

The variability of the spectral properties of hydrologic and climatic data may be in the form of either a change in the shape of the spectral density function of the background continuum or a change in either the amplitudes or frequencies of the added line components, or both. Since the detection of such type of nonstationarity requires both time and frequency information simultaneously, neither the time nor the frequency representation of the signal is sufficient for the analysis.

The basic theory for the spectral analysis of nonstationary time series was developed by Priestley (1965) under the name of "Evolutionary Spectral Analysis". The evolutionary spectrum of a nonstationary time series is time-dependent and describes the local power–frequency distribution at each instant in time.

The numerical computation of evolutionary spectra is roughly equivalent to splitting up a realization in several segments and then estimating individual spectra for each of the segments. Priestley (1965) suggests a method for calculating the evolutionary spectra of a nonstationary time series based on a double windowing

technique. In this paper the MTM method (Thomson, 1982) is used for evaluating evolutionary spectra of the analysed hydrologic and climatic data. The details of the technique are given in Hamed (1997).

Priestley & Subba Rao (1969) suggest a test for nonstationarity in time series based on evolutionary spectra. The test has the form of the analysis of variance for a two-factor design, based on the logarithms of the evolutionary spectral estimates. The details of a modified test which uses the MTM spectral analysis method is given in Hamed (1997).

The data used in this study consist of 26 US Geological Survey (USGS) flow stations in addition to 36 Historical Climatology Network (HCN) rainfall and temperature stations from the midwest USA.

EVOLUTION OF LINE SPECTRA IN HYDROLOGIC AND CLIMATIC DATA

Figure 1(a) shows the evolution of line components as well as their significance in the USGS flow data from the Mississippi River at Clinton, Iowa, station ($N = 119$ years, 1874–1992). A sliding window of length 60 years is applied at 2-year increments to produce the evolutionary spectra. The size of the circles indicates the significance of components in excess of 90%, and the plus sign indicates a $100(1 - 1/N)\%$ significance. Figure 2(a) shows the evolution of the amplitudes of line components for the same time series. The frequencies at the far right are those for the whole time series. In these Figures, two main features can be identified. The first feature is the drift in the frequency of a number of components such as the 4.51-year and the 11.3-year components. The second feature is the apparent aggregation and then splitting of components especially at low frequencies.

Figure 1(b) shows the evolution of the frequencies and significance of line components in the HCN temperature data from the Upper Sandusky, Ohio, station ($N = 107$ years, 1881–1987) using a sliding window of length 60 at 2-year increments. Figure 2(b) shows the corresponding evolution of the magnitude of line components. Frequency drifts are also clear in Fig. 2(b), especially at the 3.95-year period. Also, an obvious feature is the aggregation of the 2.5-year and 2.8-year periods into a period of about 2.6 at segment 18.

Figure 1(c) shows the evolution of the frequencies and significance of line components in the HCN rainfall data from the Griggsville, Illinois, station ($N = 114$ years, 1874–1987) using a sliding window of length 60 at 2-year increments. Figure 2(c) shows the corresponding evolution of the magnitude of line components. In Figs 1(c) and 2(c), the most stable component is that at about 3.3 years. Frequency drifts can be observed at low frequencies as well as high frequencies.

In general, the results in Figs 1 and 2 show some similarities in terms of frequency drifts and the aggregation and splitting at a number of frequencies. This behaviour is typical of nonlinear systems. It should be noted that there are no available statistical tests for assessing the significance of such changes as is the case with continuous spectra. However, there is a strong indication that the behaviour of the analysed time series is atypical of time series generated by stationary linear systems.

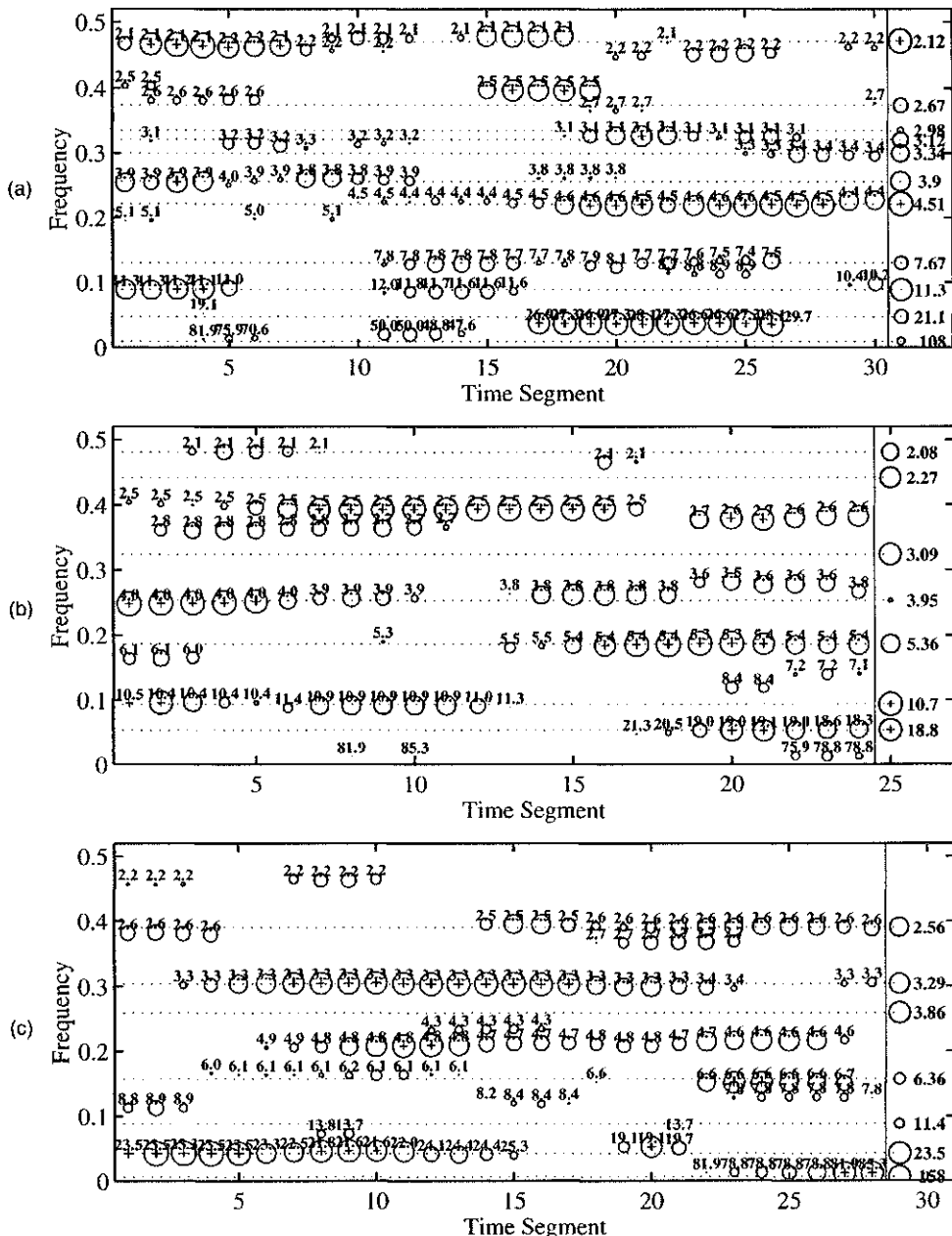


Fig. 1 Evolution of line components in (a) flow, (b) temperature, and (c) rainfall data.

VARIABILITY OF CONTINUOUS SPECTRA

The results of testing the USGS flow data series for stationarity by using the Priestley & Subba Rao (1969) test indicate that 13 of the time series are nonstationary at the 0.05 level, and 11 are nonstationary at the 0.01 level. For this group of 26 stations, the

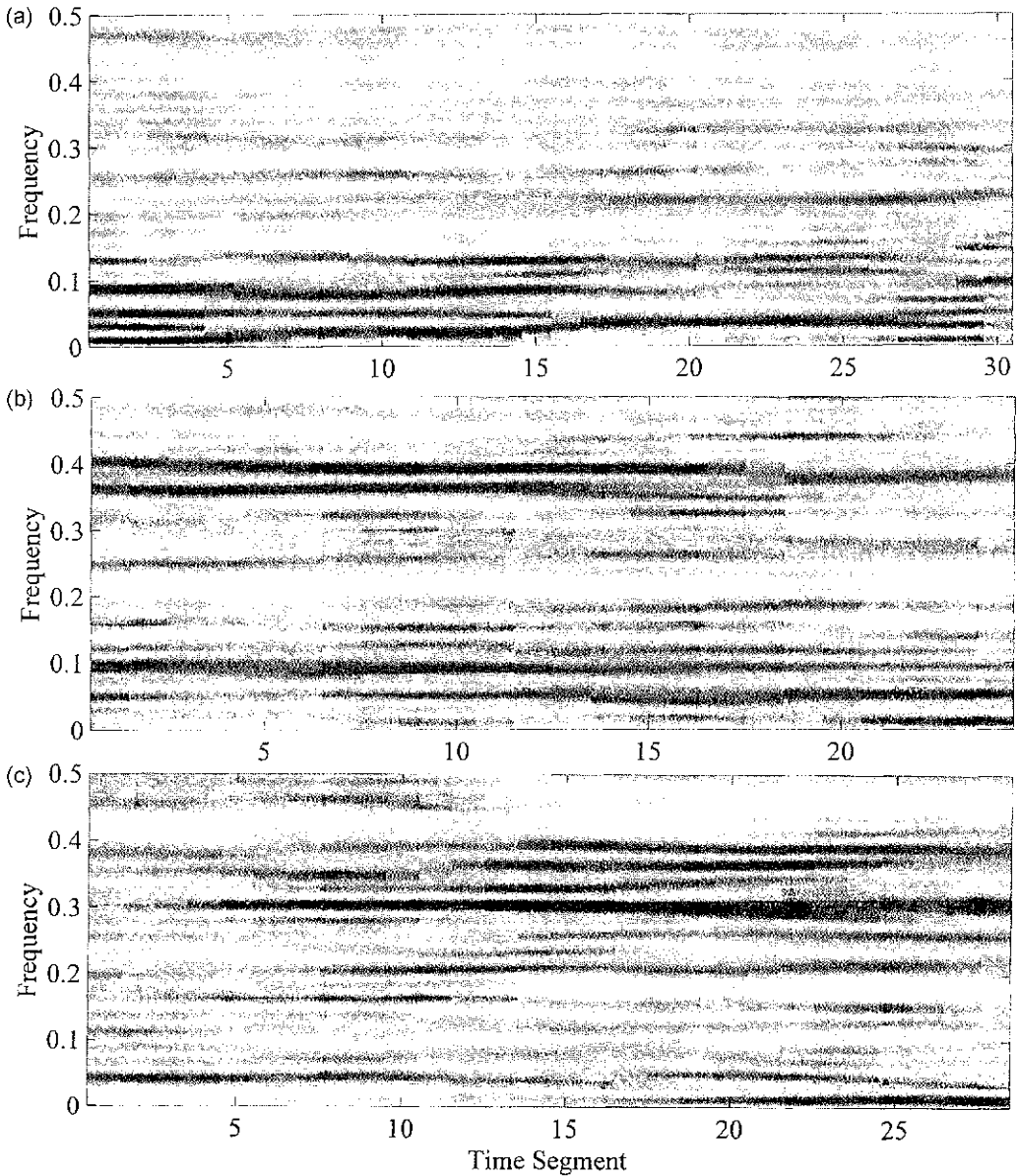


Fig. 2 Evolution of the spectral amplitude in (a) flow, (b) temperature, and (c) rainfall data.

95% confidence interval for the number of stations giving false indication of nonstationarity would be 0–3 at the 0.05 level, and 0–2 at the 0.01 level. Therefore, the number of nonstationary time series in this group is larger than would be due to chance.

The results for the HCN rainfall data indicate that 11 time series are nonstationary at the 0.05 level, and six time series are nonstationary at the 0.01 level. For this data set of 36 stations, the 95% confidence interval for the number of stations giving false indication

of nonstationarity is 0–4 at the 0.05 level, and 0–2 at the 0.01 level. Based on these numbers, there is some evidence of nonstationarity in the HCN rainfall data set.

The results of testing the HCN temperature data for stationarity indicate that 19 time series are nonstationary at the 0.05 level, and eight time series are nonstationary at the 0.01 level. For this data set of 36 stations, the 95% confidence interval for the number of stations giving false indication of nonstationarity is 0–4 at the 0.05 level, and 0–2 at the 0.01 level. This would indicate nonstationarity in the HCN temperature data.

CONCLUSIONS

The results of the analysis of both components of the mixed spectra agree in that the analysed hydrologic and climatic time series have spectra that are changing with time. This would indicate that the processes generating such time series are nonlinear systems. Nonlinear dynamical systems have the ability to undergo bifurcations as a result of the change of one of their parameters (Ghil & Childress, 1987). Therefore, the use of nonlinear models may be more efficient in the modelling of hydrologic and climatic time series. This will, hopefully, translate into more accurate forecasting of such time series.

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