

## **Fractal characteristics of drainage basin boundaries in Puerto Rico**

**SCOTT RICE-SNOW**

*Department of Geology, Ball State University, Muncie, Indiana 47306, USA*

**Abstract** Fractal dimension ( $D$ ) values ranging from 1.06 to 1.11 characterize the mapped boundaries of six of the largest drainage basins in Puerto Rico (drainage areas 186–538 km<sup>2</sup>). These  $D$  values from tropical landscapes fit within the range of fractal dimension (1.06–1.12) obtained for twelve drainage basins in the conterminous United States, supporting the hypothesis that the plan-view irregularity of drainage divides is fairly constant between tropical and temperate climatic regimes. Self-similar wandering of divides at scales finer than the resolution of available maps or other remotely-collected topographic data introduces uncertainty in the precise mapping of drainage basin boundaries for water resources regulation and management. The degree of this uncertainty is determined not only by mapping resolution, but also by basin boundary fractal dimension. Results of this study therefore suggest similar constraints on the accuracy of drainage divide mapping for the two climatic regimes.

### **Características fractales de los márgenes de las cuencas de los ríos en Puerto Rico**

**Resumen** Los valores de la dimensión fractal ( $D$ ) que varían entre 1.06 y 1.11, caracterizan los límites indicados en el mapa de seis de las más grandes cuencas de río (áreas que oscilan entre 186 y 538 km<sup>2</sup>) en Puerto Rico. Estos valores  $D$  de paisajes tropicales están dentro de los límites de la dimensión fractal (entre 1.06 y 1.12), obtenidas para doce cuencas de río en el vecino Estados Unidos, apoyando la hipótesis de que la irregularidad de plano visual de las divisorias de drenaje es bastante constante entre los regímenes climáticos tropicales y los templados. Los contornos auto-similares (aquellos que siguen el mismo esquema a escalas diferentes) de las divisorias a escalas más definidoras que la resolución de los mapas disponibles o otros datos topográficos tomados a distancia introducen ambigüedad en la cartografía precisa de las cuencas para la regulación de los recursos acuíferos y su control. El grado de esta incertidumbre viene determinado no sólo por la resolución cartográfica, sino también por la dimensión fractal del límite de la cuenca. Los resultados de este estudio, por lo tanto, sugieren limitaciones en la exactitud de las divisorias de la cuenca cartografiadas para los dos regímenes climáticos.

## **INTRODUCTION**

### **Fractal character of drainage basin boundaries**

The irregular curves of drainage basin boundaries (map-view projections of drainage divides) can be investigated as fractal geometric forms. Previous work on twelve drainage basins in the conterminous United States (Breyer & Snow, 1992) has shown that while basins differ in overall shape, the irregularity of their boundaries is quite

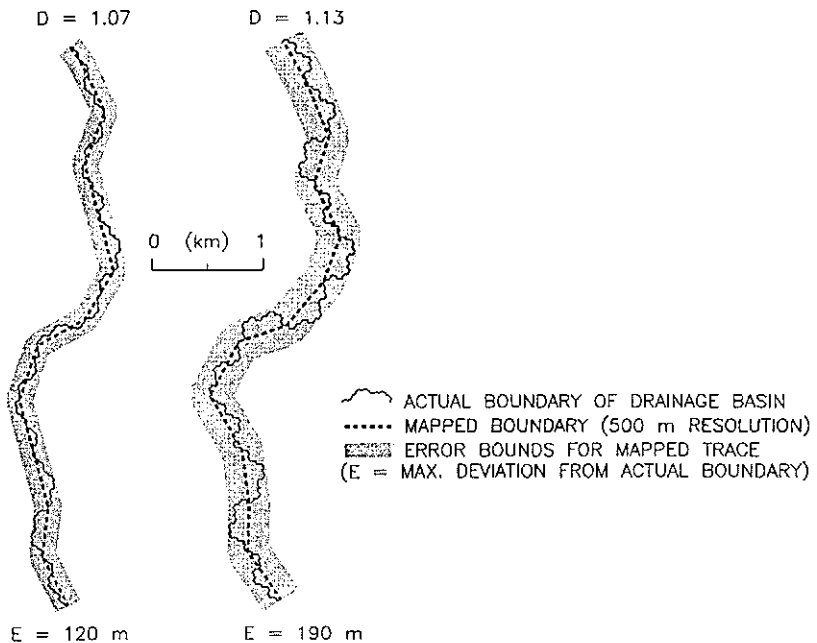
consistent. This is documented by a narrow range of fractal dimension ( $D$ ) values, 1.06–1.12, for basins developed under various geologic and climatic conditions.

Breyer & Snow (1992) use this fractal view of basin boundary irregularity and associated indeterminacy of perimeter value to suggest changes in practice for drainage basin morphometry, with resulting implications for flood prediction and control.

### Other practical significance of boundary fractal dimension

A fractal model for basin boundary form suggests that both very large and very small segments of the boundary trace, if viewed at their different, appropriate scales, will exhibit similar degrees of wandering. We can expect this fractal wandering of the basin boundary trace to be limited in scale range, ending with a smoothed trace at hilltop scales small enough to show the dominance of “diffusive” soil movement processes over erosive land-sculpting processes such as soil wash (Carson & Kirkby, 1972; Tarboton *et al.*, 1992).

However, the mapping of drainage basin boundaries is typically carried out at scales of minimum resolution much coarser than the *diffusive scale limit*, resulting in an artificial generalization and smoothing of the boundary traces. This minimum resolution may relate to the actual data resolution of remotely-sensed images or digital terrain models, ambiguities due to vegetation cover on aerial photographs, or



**Fig. 1** Conceptual diagram showing limited accuracy in mapping of drainage basin boundaries that are fractal in form. Fractal curves displayed as example basin boundary segments are varieties of Koch curves (Mandelbrot, 1983, p. 46) having fractal dimension ( $D$ ) values of 1.07 and 1.13, constructed with, respectively,  $135^\circ$  and  $120^\circ$  angles between adjoining elements.

contour spacing on topographic maps. The mapped basin boundary trace diverges from the actual boundary by not following its minor twists and turns (Fig. 1).

This divergence adds uncertainty to water resource management of events or practices near drainage divides, where the precise location of a point source relative to a divide determines whether basin water quality will be effected. Mapped basin boundaries might reasonably be accompanied by error bounds, zones within which a lack of field checking may result in improper attribution of effects to one drainage basin or another.

The fractal character of basin boundaries makes the size of such error bounds predictable. It depends on the coarseness of initial mapping, but is also significantly affected by the fractal dimension of the basin boundary in question (Fig. 1). The narrow range of fractal dimensions for basin boundaries in the conterminous US (Breyer & Snow, 1992) limits this latter influence on error bounds.

### Objectives of this study

The present study is an application of the same analysis techniques to a set of drainage basins in the humid tropics. It provides a test of the general hypothesis that drainage basin boundaries have a consistent geometry, well expressed by a limited range of fractal dimension ( $D$ ) values, unaffected by geologic and climatic controls. It also gives a preliminary basis for practical application of this geometric understanding to water resources management in the humid tropics.

## METHODS

### Data collection

Six major drainage basins in the uplands of Puerto Rico were chosen for analysis (Fig. 2). The basins, all of which terminate at river gauging stations, were selected to be as large as possible, allowing inspection of boundary irregularities across a variety of scales. However, it was also important to select basins entirely outside the

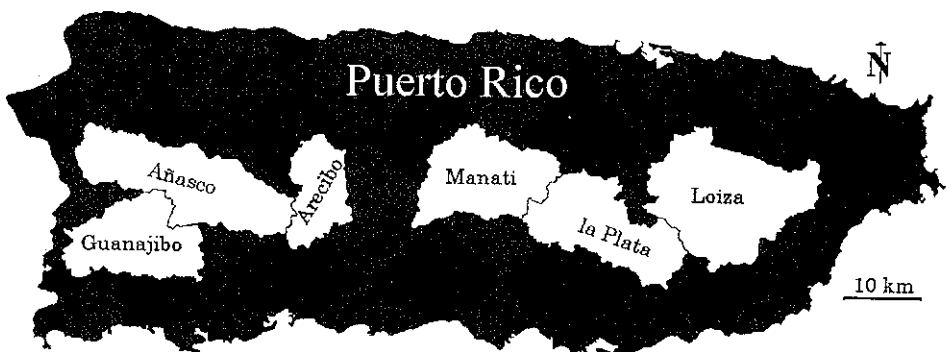


Fig. 2 The six major drainage basins investigated in this study.

island's east-west-trending, northern karst belt, a region where determination of basin boundaries becomes problematic, and to limit basin contiguity so that each basin boundary would remain largely unique in its form.

The basins range from 186 to 538 km<sup>2</sup> in area (Table 1). Although Puerto Rico exhibits much local climatic variability, basin-scale spatial climatic averages place all of the selected basins within the Humid Tropical morphoclimatic zone (Summerfield, 1991). Inspection of within-basin variations in mean annual precipitation and temperature and consideration of alternative morphogenetic classifications (Wilson, 1968, 1969) suggest a gradation in degree of Humid Tropical climate characteristics, with Añasco and Arecibo basins being most strongly within the humid tropical regime; Guanajibo, Manatí, and Loíza basins being more peripherally so; and La Plata basin fitting only marginally, verging on the Tropical Wet-Dry regime.

The six drainage basins exhibit some differences in underlying geology within a general igneous-metamorphic framework, and two have boundary segments strongly associated with regional fault traces. Puerto Rico is in a tectonically active area, an additional contrast with the areas treated by Breyer & Snow (1992) in terms of geologic controls.

Drainage basin boundary traces were digitally compiled from an ARC/INFO coverage of drainage divides developed by the GIS Unit of the Caribbean District

**Table 1** Six major drainage basins of Puerto Rico, their climatic and geologic characteristics, and boundary trace fractal dimension values (estimated by the divider method).

Drainage basin (with defining location)*	Drainage area (km <sup>2</sup> )	Climate (mean annual values)+	Geology	Fractal dimension ( <i>D</i> ) of basin boundary
Río Guanajibo at station 50-13800, near Hormigueros	311	Prec. 210 cm year <sup>-1</sup> Temp. 24°C	volcaniclastics, serpentinite	1.10
Río Grande de Añasco at station 50-146070, near Añasco	465	Prec. 230 cm year <sup>-1</sup> Temp. 23°C	volcaniclastics, serpentinite	1.06
			<i>subdivisions:</i> fault control	(1.05)
			no fault control	(1.07)
Río Grande de Arecibo at station 50-025000, near Utuado	186	Prec. 210 cm year <sup>-1</sup> Temp. 23°C	volcaniclastics, serpentinite, granodiorite	1.10
Río Grande de Manatí at station 50-035000, at Ciales	348	Prec. 210 cm year <sup>-1</sup> Temp. 22°C	volcaniclastics	1.10
Río de la Plata at station 50-044000, near Comerio	360	Prec. 160 cm year <sup>-1</sup> Temp. 22°C	Volcaniclastics	1.11
Río Grande de Loíza at station 50-058800, Lago Loíza dam	538	Prec. 205 cm year <sup>-1</sup> Temp. 24°C	Volcaniclastics, granodiorite	1.09
			<i>subdivisions:</i> fault control	(1.03)
			volcaniclastics	(1.11)
			granodiorite	(1.08)

\* Gauging station identification numbers and drainage areas obtained from Curtis *et al.* (1985).

+ Spatial averages for basin areas, estimated from 1931-1960 data (Calvesbert, 1970).

Office, Water Resources Division, US Geological Survey. The traces represent maximum accuracy in divide definition available from 1:20 000 scale topographic maps.

### Divider method of fractal analysis

Divider analysis (Mandelbrot, 1983) provides a statistical summary of the geometry of an irregular curve in two-dimensional space, showing the degrees of curve wandering to be found at various scales of observation. The length of the curve is repeatedly walked by a real or virtual map divider, giving a suite of estimated curve lengths, each keyed to a particular step size (or resolution of measurement). If a doubly-logarithmic plot of estimated curve length vs step size is linear, this indicates a self-similar fractal geometry for the curve, with fractal dimension ( $D$ ) being a function of the plot slope.

For this study, digital analysis of the basin boundary data employed the same algorithm used by Breyer & Snow (1992). Divider walks originated from and ended at the basin mouths. The minimum step size was 300 m, a scale below which mapping generalization is likely to smooth the drainage basin boundary drawn from the 1:20 000 contour maps. The maximum step size was calculated from drainage area,  $A$ , to be  $0.1A^{0.5}$  (Breyer & Snow, 1992), roughly 2000 m for these basins, eliminating larger scales of view in which the overall basin shape, rather than boundary irregularity alone, affects the analysis results. For the two largest basins, Añasco and Loíza, segments of the basin boundaries distinguished by differing geologic controls (Fig. 3) were also subjected to separate analyses.

## RESULTS

Fractal character was confirmed for the six basin outlines. The derived estimates of fractal dimension for basin boundaries, listed in Table 1, range from 1.06 to 1.11.

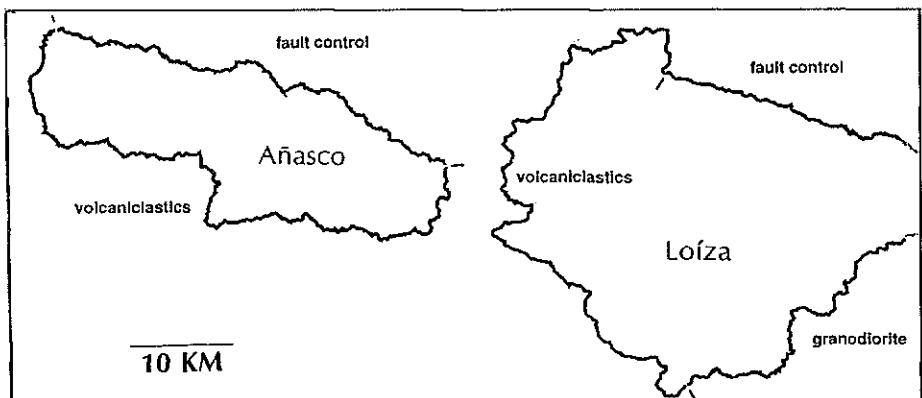


Fig. 3 Subdivisions of the boundaries of Rio Grande de Añasco and Rio Grande de Loíza basins, marked by differing geologic controls.

The drainage boundary subdivisions for Añasco and Loíza basins (Fig. 3, with  $D$  values listed in parentheses in Table 1) in each case explicitly separate out a boundary segment that is notably linear at large scale of view, correlating with a major mapped fault zone. These segments distinguished by fault control yield unusually low fractal dimension values for finer scales of inspection, to the 300-m resolution limit of analysis.

A consistent influence of climate or rock type on the fractal dimension describing basin boundary irregularity is not demonstrated within this small data set. Patterns that may appear to arise, do not bear out uniformly. A decrease in  $D$  with wetter climate, supported by the extreme values in Table 1, is not supported by a gradation of the intermediate values. Although basin boundaries underlain completely by volcanoclastics (including a boundary segment of the Loíza basin) display uniformly high  $D$  values, those underlain by a mix of volcanoclastics and serpentinite (including the segment of the Añasco basin boundary without fault control) have inconsistent values.

Furthermore, comparison between these results and those obtained for the conterminous United States (Breyer & Snow, 1992) suggests no such trends related to their much greater differences in climate and rock type. The  $D$  values obtained for Puerto Rico, with its tropical climate, dominantly igneous terrain, and more active tectonic environment, fit within the same range obtained in the earlier study.

## CONCLUSIONS

These results support the hypothesis that drainage basin boundaries have consistent degrees of irregularity, as expressed by a narrow range of fractal dimension values (1.06–1.12), regardless of prevailing climate or underlying rock type. Anomalously straight boundary segments associated with regional fault lines appear to display a similarly smoothed geometry at fine scales (less than 1 km resolution), indicated by  $D$  values of 1.05 or less.

The self-similar irregularity characteristics of drainage basin boundaries in tropical and temperate regions appear to be indistinguishable. The same practical outcomes of the consistent fractal character of basin boundaries for basin morphometry (Breyer & Snow, 1992) and for water resources management near divides (discussed above) apply equally well in both climatic regimes.

### Need for further work

Field investigations are necessary to confirm that the wanderings of drainage divides at fine scales (yet still above the diffusive scale limit) continue the self-similar geometry observed at scales apparent on topographic maps. These investigations must encompass both temperate and tropical examples. Work is also needed to better constrain estimations of maximum error bound values for delineation of basin boundaries, as a function of boundary fractal dimension, mapping resolution, and diffusion scale limit.

**Acknowledgements** Raw basin boundary data for this study were provided by the GIS Unit of the Caribbean District Office, Water Resources Division, US Geological Survey, San Juan, Puerto Rico. I am grateful to R. Webb, A. Sepulveda, J. Troester, and L. Torres of the USGS for technical advice and assistance. J. Emert constructed the example fractal curves (Fig. 1), and J. Rice-Snow and R. Fuertes Manjón provided Spanish translation of the abstract.

## REFERENCES

- Breyer, S. P. & Snow, R. S. (1992) Drainage basin perimeters: a fractal significance. *Geomorphol.* **5**, 143-157.
- Carson, M. A. & Kirkby, M. J. (1972) *Hillslope Form and Process*. Cambridge Univ. Press, Cambridge.
- Calvesbert, R. J. (1970) Climate of Puerto Rico and US Virgin Islands. *Climatography of the United States no. 60-52, US Dept of Commerce, Silver Spring, Maryland, USA*.
- Curtis, R. J., Guzmán-Ríos, S. & Díaz, P. L. (1985) Water resources data, Puerto Rico and the US Virgin Islands, water year 1984. *USGS Water-Data Report PR-84-1, Dept of Interior, San Juan, Puerto Rico*.
- Mandelbrot, B. B. (1983) *The Fractal Geometry of Nature*. Freeman & Co., New York.
- Summerfield, M. A. (1991) *Global Geomorphology*. Longman Scientific & Technical, Singapore.
- Tarboton, D. G., Bras, R. L. & Rodriguez-Iturbe, I. (1992) A physical basis for drainage density. *Geomorphol.* **5**, 59-76.
- Wilson, L. (1968) Morphogenetic classification. In: *Encyclopaedia of Geomorphology* (ed. by R. W. Fairbridge), Hutchinson & Ross, Dowden, UK.
- Wilson, L. (1969) Relationships between geomorphic processes and modern climates as a method in paleoclimatology. *Revue de Géographie Physique et de Géologie Dynamique* **11**, 303-314.